

# Workshop on the Theory of Belief Functions

April 1-2, 2010

Brest, France

## Tutorial

March 31, 2010

16h30

### Formal representations of uncertainty

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The modeling of uncertainty is motivated by two concerns: taming the variability of external phenomena and facing incomplete information in decision processes. These two concerns are not unrelated, but the two concerns are distinct in the sense that variability is far from being the only cause of ignorance. However, the development of probability theory, as witnessed by the Bayesian school especially, tended to blur this distinction, suggesting that a unique probability distribution is enough to account for both randomness and incomplete information. More recently, new theories of uncertainty have emerged where partial ignorance is acknowledged and represented separately from randomness:

the theories of evidence, possibility and imprecise probabilities, respectively. The aim of this talk is to provide a (partially) unified view of these approaches. The main point of the talk is that modern uncertainty theories put together probabilistic and set-valued representations, which allow for a clear separation between randomness and incompleteness.

The basic tool for representing information incompleteness is a subset of mutually exclusive values, one of which is the real one. This kind of uncertainty is naturally accounted for in logical representations.

In the area of numerical modeling, the processing of incomplete information is basically carried out by interval analysis or constraint propagation methods. All above theories of uncertainty come down to introducing shades of plausibility within set-representations of incompleteness:

- in imprecise probability theory, the most general one, information takes the form a set of probability measures, all the larger as information is poor.
- in evidence theory, information is represented by random sets, which correspond to special subsets of probabilities.
- in possibility theory, information is summarized by fuzzy sets or fuzzy intervals, then equivalent to nested random sets.

The course points out some important issues to be addressed with uncertainty theories such as the difference between generic and singular information, practical representations of imprecise probabilities on the real line, conditioning and fusion, uncertainty propagation, and decision. The role of possibility theory, as the simplest representation of imprecise probability will be emphasized.

