# Audit Risk Formula with Mixed Evidence 

Rajendra P. Srivastava ${ }^{1}$, and Theodore J. Mock ${ }^{2}$<br>${ }^{1}$ School of Business, The University of Kansas, Lawrence, Kansas 66045<br>Email: rsrivastava@ku.edu<br>${ }^{2}$ Anderson Graduate School of Management, University of California, Riverside, CA and University of Maastricht, The Netherlands<br>Email: tmock@ucr.edu


#### Abstract

The financial statement audit is the process of collecting, evaluating, and aggregating relevant items of evidence pertaining to various management assertions related to the financial statement accounts to determine whether the company's financial statements present fairly its financial position. The Dempster-Shafer theory [1] of belief functions has been argued to be an appropriate framework for representing uncertainties in the audit. This note extends Srivastava and Shafer [2] by deriving an audit risk formula under the Dempster-Shafer theory for more realistic situations where the auditor has mixed items of evidence. In their derivation, Srivastava and Shafer assume only positive evidence. In addition, their work is extended by considering interrelationships among the balance sheet accounts and the transaction streams accounts. Such interrelationships which are prominent in practice, were not considered by Srivastava and Shafer.


Key Words: Financial Statement Audit, Audit Risk, Audit Planning and Evaluation, Dempster-Shafer Theory, Belief Functions

## I. INTRODUCTION

The financial statement audit of a company is the process of collecting, evaluating, and aggregating relevant items of evidence pertaining to various management assertions related to the balance sheet accounts and the corresponding transaction stream (income statement) accounts to determine whether the company's financial statements present fairly its financial position and financial performance. The auditing profession world-wide has developed auditing standards to help auditors achieve the above goal. In the process, the audit profession in each country has not only developed standards as to what kinds of evidence should be gathered but also has provided a model to aggregate various items of evidence gathered in the audit process. For example, the American Institute of Certified Public Accountants (AICPA) in the USA has provided the following audit risk model [3]: AR = IR.CR.AP.DR, for assessing audit risk. AR is the risk that the auditor has failed to detect material misstatements relevant to an assertion ${ }^{1}$ pertaining to either an account

[^0]balance, a class of transactions, or a disclosure, and issued an opinion that the financial statements are fairly presented.
The AICPA audit risk model suggests that audit risk is the product of four risks:

- IR, the inherent risk that a material misstatement associated with an assertion is present due to inherent nature of the account, class of transactions, or the disclosure.
- CR, the control risk that internal accounting controls has failed to prevent or detect and correct a material misstatement relevant to the assertion.
- AP, the analytical procedures risk that analytical procedures have failed to detect material misstatements relevant to the assertion and
- TD, the test-of-details risk that the audit procedures have failed to detect material misstatements relevant to the assertion.
While the above audit risk model provides a way to assess the risk of material misstatement pertaining to an assertion, it does not provide an appropriate way to aggregate items of evidence for various accounts and transaction streams constituting the financial statements. The overall aggregation process is left on the auditor's professional judgment.
Srivastava and Shafer [2] present analytical formulas under Dempster-Shafer (DS) theory [1] which combine items of evidence at three levels: the accounts in the balance sheet, their respective assertions, and at the overall financial statement level. They demonstrated that the AICPA audit risk model may be interpreted as a plausibility model under DS theory where each risk term in the model represents the plausibility that material misstatement is present in the assertion being considered in the model. For example, CR may be interpreted as the plausibility that material misstatement is not prevented or detected and corrected by internal accounting controls relevant to each assertion.
Although the general evidential diagram for the audit process is a network [5], Srivastava and Shafer used a tree type evidential diagram to develop the analytical formulas for audit risk and considered only affirmative

[^1]items of audit evidence, that is evidence that supports the variable being evaluated. Also, they did not consider the interrelationships between the accounts in the balance sheet and the transaction streams although it is clear from the nature of double-entry accounting systems that these are interrelated. Thus, consideration of such interrelationships is important for developing a more comprehensive analytical formula for assessing the overall audit risk. Auditing standards [3, 4] also suggest such interrelationships exist in practice.

In the present paper, we extend Srivastava and Shafer [2] by developing an audit risk formula, which incorporates mixed items of evidence and the interrelationships among the accounts in the balance sheet and the transaction streams. While the Srivastava and Shafer models are useful for planning purposes, our model is useful for both planning and evaluation purposes. For the planning phase, Srivastava and Shafer have argued that the auditor assumes positive items of evidence. However, given that both negative and mixed items of evidence may be encountered in an audit, a framework that incorporates both types of evidence may be used for both planning and evaluation. Srivastava et al [5] have developed a fraud risk assessment model that considers mixed evidence.

## II. AUDIT PROCESS AND EVIDENTIAL DIAGRAM

In a typical audit, the auditor has evidence at various levels of the financial statements:

- Evidence at the overall financial statements level such as information concerning management integrity and the economic environment,
- Evidence at the individual balance sheet account level such as industry norms for the balances,
- Evidence at the transaction streams or class of transactions level such as ratio analyses of sales and cash receipts which are related to the accounts receivable balance in the balance sheet
- Evidence at the assertion level such as physical counts of inventory for the assertion that inventory exists and is valued correctly and
- Evidence related to disclosure requirements at the overall financial statement level or individual account and transaction stream level such as evidence about the sale of an important subsidiary after the year-end but before the completion of the audit.


## A. Evidential Diagram and the Audit Process Logic

An evidential diagram of an audit can be represented as a network of variable nodes [6] where the variables include the overall financial position of the company (i.e. the balance sheet), the individual asset, liability and equity accounts which constitute the balance sheet, the management assertions associated with these accounts, and the corresponding classes of transactions and their management assertions. In Figure 1, the rectangular boxes with rounded corners represent the variable nodes and the circles with the symbol ' $\&$ ' represent the 'AND' relationship between the
variable on the left with the variables on the right. These variables represent the various assertions, as given in Table 1, for which the auditor collects evidence to ascertain that they are true and ultimately to make the judgment as to whether the overall financial statements of the company are free or are not free from material misstatements.

## Table 1

Management assertions based on AICPA [3] with modifications to make the assertions generic to an account on the balance sheet.

| Main Assertions | Sub Assertions related to an Account or Class of Transactions |
| :---: | :---: |
| The ith Account <br> (Ai) in the <br> Balance Sheet is free from material misstatements. $\mathrm{i} \in\{1,2,3, \mathrm{n}\}$ | Ai.1: Existence. The ith balance sheet account (asset, liabilities, or equity) exists, that is it is not fictitious. |
|  | Ai.2: Rights and obligations. For the ith account, the entity holds or controls the rights to an asset account, or has the obligation for a liability account. |
|  | Ai.3: Completeness. All assets, liabilities, and equity interests relevant to ith account that should have been recorded have been recorded. |
|  | Ai.4: Valuation and allocation. The ith account is included in the financial statements at the appropriate amount and any resulting valuation or allocation adjustment is appropriately recorded. |
| The kth Class of transactions and events (CAi.j.k) for the period under audit related to assertion Ai.j is free from material misstatement. $\mathrm{j} \in\{1,2,3,4\}$ <br> and $\mathrm{k} \in\{1,2,3, \mathrm{qij}\}$ where $\mathrm{q} i \mathrm{j}$ is the total number of classes of transactions associated with the assertion Ai.j. | CAi.j.k.1: Occurrence. The kth class of transactions and events related to assertion Ai.j that have been recorded have occurred and pertain to the entity. |
|  | CAi.j.k.2: Completeness. All transactions and events related to kth class of transactions and events pertaining to assertion Ai.j that should have been recorded have been recorded. |
|  | CAi.j.k.3: Accuracy. Amounts and other data relating to recorded kth transactions and events pertaining to assertion Ai.j have been recorded accurately. |
|  | CAi.j.k.4: Cutoff. The kth class of transactions and events related to assertion Ai.j have been recorded in the correct accounting period. |
|  | CAi.j.k.5: Classification. The kth class of transactions and events relevant to Ai.j have been recorded in the proper accounts. |
| The presentation and disclosures related account $\mathbf{A i}$ (DAi) are appropriate. | DAi.1: Occurrence and rights and obligations. Disclosed events and transactions relevant to account Ai have occurred and pertain to the entity. |
|  | DAi.2: Completeness. All disclosures that should have been included related to account Ai have been included. |
|  | DAi.3: Classification and understandability. Financial information related to account Ai is appropriately presented and described and disclosures are clearly expressed. |

All variables in Figure 1 are binary variables. We use the upper case letters to represent the names of the variables, and the lower case letters for their values. For example, the variable $\mathbf{B}$ represents the assertion that the overall balance sheet of the company is free from material misstatements. B has two possible values, ' $b$ ' and ' $\sim b$ ' where ' $b$ ' represents the state where the assertion $\mathbf{B}$ is true and ' $\sim b$ ' represents the state where the assertion is not true. Consistent with Srivastava and Shafer [2], we assert that the balance sheet is fairly stated if and only if all the accounts in the balance sheet are free from material misstatements. In addition, we assert that all the disclosures at the overall level and at the account level must also be appropriate, that is consistent with the standards of disclosure, which have recently been added to the auditing standards [4]. The relationship between an account balance and the corresponding classes of transactions is represented by the 'AND' relationship as depicted in Figure 1.
In summary, the Figure 1 evidential diagram asserts that a balance sheet account, say Ai , is free from material misstatements if and only if all its assertions (Ai.1, Ai.2, Ai.3, and Ai.4) are true and the corresponding disclosures are made properly. Also, the $\mathrm{j}^{\text {th }}$ assertion of account Ai is true if and only if all the related classes of transactions are free from material misstatements. All these relationships are depicted through the 'AND' tree depicted in Figure 1. Because of lack of space, we have not depicted the various items of evidence pertaining to each variable in the network. However, we discuss these items of evidence in brief in the next section.

## B. Items of Evidence and Corresponding Belief Masses (mvalues)

In general, the auditor obtains multiple items of evidence pertaining to each variable in Figure 1. We depict an item of evidence with letter E with a subscript that indicates the corresponding variable. For simplicity, let us denote a variable node by X with values $\{\mathrm{x}, \sim \mathrm{x}\}$ and the corresponding items of evidence directly bearing on that variable by $\mathrm{E}_{\mathrm{pX}}$, where the subscript pX represents $\mathrm{p}^{\text {th }}$ item of evidence directly bearing on X . The corresponding belief masses are represented by symbols, $\mathrm{m}_{\mathrm{pX}}^{+}=\mathrm{m}_{\mathrm{p}}(\mathrm{x}), \mathrm{m}_{\mathrm{pX}}^{-}=$ $m_{p}(\sim x)$ and $m_{p X}^{\Theta}=m_{p}(\{x, \sim x\})$, respectively, representing the belief masses, that is the strength or weight of the evidence, in support of the assertion $x$, in support of the negation of the assertion $\sim \mathrm{x}$, and the residue level of ambiguity, such that $\mathrm{m}_{\mathrm{pX}}^{+}+\mathrm{m}_{\mathrm{pX}}^{-}+\mathrm{m}_{\mathrm{pX}}^{\Theta}=1$.

## III. AUDIT RISK FORMULA AT THE BALANCE SHEET LEVEL

To derive the analytical formula for audit risk at the overall balance sheet level, we propagate all the belief masses from all the variable nodes in the evidential diagram given in Figure 1 to the main variable $B$. For this purpose, we complete the following steps.

Step 1: Assessing the Belief Masses at Each Variable
Since we are considering multiple items of evidence for each binary variable in the evidential network, we use Dempster rule as simplified by Srivastava [7] to determine the overall belief masses directly defined at each variable by combining all evidence, say $q_{X}$ of them, directly bearing on the variable. We obtain the following belief masses in support of $\mathrm{X}, \mathrm{m}_{\mathrm{x}}^{+}$, and against $\mathrm{X}, \mathrm{m}_{\mathrm{x}}^{-}$:
$\mathrm{m}_{\mathrm{X}}^{+}=1-\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X}}}\left(1-\mathrm{m}_{\mathrm{i}}(\mathrm{x})\right) / \mathrm{K}_{\mathrm{x}}$,
$\mathrm{m}_{\mathrm{x}}^{-}=1-\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X}}}\left(1-\mathrm{m}_{\mathrm{i}}(\sim \mathrm{x})\right) / \mathrm{K}_{\mathrm{x}}$,
$\mathrm{m}_{\mathrm{X}}^{\Theta}=\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X}}} \mathrm{m}_{\mathrm{i}}\left(\{\mathrm{x}, \sim \mathrm{x}) / \mathrm{K}_{\mathrm{X}}\right.$.
where $K_{x}$ is the renormalization constant and is given by
$\mathrm{K}_{\mathrm{x}}=\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X}}}\left(1-\mathrm{m}_{\mathrm{i}}(\mathrm{x})\right)+\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X}}}\left(1-\mathrm{m}_{\mathrm{i}}(\sim \mathrm{x})\right)-\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X}}} \mathrm{m}_{\mathrm{i}}(\{\mathrm{x}, \sim \mathrm{x}\})$.

## Step 2: Propagation of Beliefs from Level 5 to Level 4

We use Srivastava, Shenoy, and Shafer [8] Proposition 1 to propagate the beliefs defined at the variables in level 5 to the variables in level 4 (see Figure 1). According to Proposition 1, the belief masses propagated from the variables at Level 5 to the variables at Level 4 can be written as:
$\mathrm{m}_{\mathrm{X} 4 \leftarrow \operatorname{all~} \mathrm{X} 5}^{+}=\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X} 5}} \mathrm{~m}_{\mathrm{X} 5}^{+}$, and $\mathrm{m}_{\mathrm{X} 4 \leftarrow \text { all } \mathrm{X} 5}^{-}=1-\prod_{\mathrm{i}=1}^{\mathrm{q}_{\mathrm{X} 5}}\left(1-\mathrm{m}_{\mathrm{X} 5}^{-}\right)$
Where X4 represents all the variables in Figure 1 at Level 4 , and X 5 represents all the variables at Level 5. For example, if $\mathrm{X} 4=$ CAi.2.k then $\mathrm{X} 5 \in\{$ CAi.2.k.1, CAi.2.k.2, CAi.2.k.3, CAi.2.k.4, CAi.2.k.5, DAi.2.k\}. Next, we combine the beliefs propagated from Level 5 to Level 4 variables with the beliefs directly bearing on these variables at Level 4 as determined in Step 1. We use Dempster rule as simplified by Srivastava [7] to combine the two sets of beliefs and obtain the following belief masses at Level 4:

$$
\begin{align*}
\mathrm{m}_{\mathrm{X} 4}^{\prime+} & =1-\left(1-\mathrm{m}_{\mathrm{X} 4 \leftarrow \operatorname{all~} \mathrm{X} 5}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 4}^{+}\right) / \mathrm{K}_{\mathrm{X} 4}  \tag{6}\\
\mathrm{~m}_{\mathrm{X} 4}^{\prime-} & =1-\left(1-\mathrm{m}_{\mathrm{X} 4 \leftarrow \operatorname{all~} \mathrm{X} 5}^{-}\right)\left(1-\mathrm{m}_{\mathrm{X} 4}^{-}\right) / \mathrm{K}_{\mathrm{X} 4}  \tag{7}\\
\mathrm{~m}_{\mathrm{X} 4}^{\Theta \Theta} & =\mathrm{m}_{\mathrm{X} 4 \leftarrow \operatorname{all~X} 5}^{\Theta} \mathrm{m}_{\mathrm{X} 4}^{\Theta} / \mathrm{K}_{\mathrm{X} 4},  \tag{8}\\
\mathrm{~K}_{\mathrm{X} 4} & =\left(1-\mathrm{m}_{\mathrm{X} 4 \leftarrow \operatorname{all~X} 5}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 4}^{+}\right)+\left(1-\mathrm{m}_{\mathrm{X} 4 \leftarrow \text { all } \mathrm{X} 5}^{-}\right)\left(1-\mathrm{m}_{\mathrm{X} 4}^{-}\right)  \tag{9}\\
\quad & -\mathrm{m}_{\mathrm{X} 4 \leftarrow \operatorname{all~} 5_{5}^{\Theta}}^{\Theta} \mathrm{m}_{\mathrm{X} 4}^{\Theta}
\end{align*}
$$

## Step 3: Propagation of Beliefs from Level 4 to Level 3

This step is similar to Step 2. We again propagate the beliefs from the variables at Level 4 to the variables at Level 3 by using Srivastava et. al [8] and then combine these beliefs with the beliefs defined at each variables at Level 3 using Srivastava [7]. Here are the resulting belief
masses at the variables at Level 3:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{X} 3}^{\prime+}=1-\left(1-\mathrm{m}_{\mathrm{X} 3 \leftarrow \text { all } \mathrm{X} 4}^{+}\right)\left(1-\mathrm{m}_{\mathrm{x} 3}^{+}\right) / \mathrm{K}_{\mathrm{x} 3},  \tag{10}\\
& \mathrm{~m}_{\mathrm{X} 3}^{\prime-}=1-\left(1-\mathrm{m}_{\mathrm{X} 3 \leftarrow \text { all } 44}^{-}\right)\left(1-\mathrm{m}_{\mathrm{x} 3}^{-}\right) / \mathrm{K}_{\mathrm{x} 3},  \tag{11}\\
& \mathrm{~m}_{\mathrm{X} 3}^{\prime \Theta}=\mathrm{m}_{\mathrm{X} 3 \leftarrow \text { all } \mathrm{X} 4}^{\Theta} \mathrm{m}_{\mathrm{x} 3}^{\Theta} / \mathrm{K}_{\mathrm{x} 3},  \tag{12}\\
& \mathrm{~K}_{\mathrm{X} 3}=\left(1-\mathrm{m}_{\mathrm{X} 4 \leftarrow \text { all } \mathrm{X} 4}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 3}^{+}\right)+\left(1-\mathrm{m}_{\mathrm{X} 3 \leftarrow \text { all } 44}^{-}\right)\left(1-\mathrm{m}_{\mathrm{X} 3}^{-}\right)  \tag{13}\\
& \quad-\mathrm{m}_{\mathrm{X} 3 \leftarrow \text { all } \mathrm{X} 4}^{\Theta} \mathrm{m}_{\mathrm{x} 3}^{\Theta},
\end{align*}
$$

where $\mathrm{X} 3 \in\{$ Ai.1, Ai.2, Ai.3, Ai.4\}. Similarly, the belief masses propagated from DAi.1, DAi.2, and DAi.3, to DAi when combined with the belief masses at DAi yields:

$$
\begin{align*}
& \mathrm{m}_{\text {DAi }}^{\prime+}=1-\left(1-\mathrm{m}_{\text {DAi }}^{+}\{\text {DAi. } 1, \text { DAi.2,DAi.3\} })\left(1-\mathrm{m}_{\mathrm{DAi}}^{+}\right) / \mathrm{K}_{\mathrm{DAi}}\right. \text {, }  \tag{14}\\
& \mathrm{m}_{\text {DAi }}^{\prime-}=1-\left(1-\mathrm{m}_{\text {DAi }}^{-}-\{\text {DAi. } 1, \text { DAi.2,DAi.3\} })\left(1-\mathrm{m}_{\text {DAi }}^{-}\right) / \mathrm{K}_{\text {DAi }},\right.  \tag{15}\\
& \mathrm{m}_{\mathrm{DAi}}^{\prime \Theta}=\mathrm{m}_{\mathrm{DAi} \leftarrow\{\text { DAi.1,DAi.2,DAi.3\} }}^{\Theta} \mathrm{m}_{\mathrm{DAi}}^{\Theta} / \mathrm{K}_{\mathrm{DAi}} \text {, }  \tag{16}\\
& \mathrm{K}_{\text {DAi }}=\left(1-\mathrm{m}_{\text {DAi } \leftarrow\{\text { DAi. } 1, \text { DAi. } 2, \mathrm{DAi} .3\}}^{+}\right)\left(1-\mathrm{m}_{\text {DAi }}^{+}\right) \\
& +\left(1-\mathrm{m}_{\text {DAi } \leftarrow \text { DAi. } 1, \mathrm{DAi} .2, \mathrm{DAi} .3\}}^{-}\right)\left(1-\mathrm{m}_{\text {DAi }}^{-}\right)  \tag{17}\\
& -\mathrm{m}_{\text {DAi }- \text { DAi.1,DAi.2,DA.3 }\}}^{\Theta} \mathrm{m}_{\text {DAi }}^{\Theta} \text {, }
\end{align*}
$$

## Step 4: Propagation of Beliefs from Level 3 to Level 2

In this step, we propagate belief from the variables at Level 3 to the variables at Level 2 again using Srivastava et al [8] and then combine these beliefs with the beliefs directly defined at the variables at Level 2 using Dempster's rule as simplified by Srivastava [7]. One obtains the following belief masses at the variables at Level 2:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{x} 2}^{\prime+}=1-\left(1-\mathrm{m}_{\mathrm{X} 2 \leftarrow \mathrm{all} \mathrm{X} 3}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 2}^{+}\right) / \mathrm{K}_{\mathrm{X} 2},  \tag{18}\\
& \mathrm{~m}_{\mathrm{X} 2}^{\prime-}=1-\left(1-\mathrm{m}_{\mathrm{X} 2 \leftarrow \text { all X3 }}^{-}\right)\left(1-\mathrm{m}_{\mathrm{X} 2}^{-}\right) / \mathrm{K}_{\mathrm{X} 2} \text {, }  \tag{19}\\
& \mathrm{m}_{\mathrm{X} 2}^{\prime \Theta}=\mathrm{m}_{\mathrm{X} 2 \leftarrow \mathrm{all} \mathrm{X} 3}^{\Theta} \mathrm{m}_{\mathrm{X} 2}^{\Theta} / \mathrm{K}_{\mathrm{X} 2} \text {, }  \tag{20}\\
& \mathrm{K}_{\mathrm{X} 2}=\left(1-\mathrm{m}_{\mathrm{X} 2 \leftarrow \text { all X } 3}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 2}^{+}\right)+\left(1-\mathrm{m}_{\mathrm{X} 2 \leftarrow \text { all X3 }}^{-}\right)\left(1-\mathrm{m}_{\mathrm{X} 2}^{-}\right)  \tag{21}\\
& -\mathrm{m}_{\mathrm{X} 2 \leftarrow \mathrm{all} \mathrm{X} 3}^{\Theta} \mathrm{m}_{\mathrm{X} 2}^{\Theta} \text {, }
\end{align*}
$$

where $\mathrm{X} 2 \in\{\mathrm{~A} 1, \mathrm{~A} 2, \ldots . \mathrm{An}\}$. Similarly, the belief masses at the variable DB at Level 2 can be obtained by propagating the beliefs from DB.1, DB.2, and DB. 3 to DB using [8] and combining these beliefs with the beliefs at DB using [7]:

$$
\begin{align*}
\mathrm{m}_{\mathrm{DB}}^{\prime+} & =1-\left(1-\mathrm{m}_{\mathrm{DB} \leftarrow\{\mathrm{DB} .1, \mathrm{DB} .2, \mathrm{DB} .3\}}^{+}\right)\left(1-\mathrm{m}_{\mathrm{DB}}^{+}\right) / \mathrm{K}_{\mathrm{DB}},  \tag{22}\\
\mathrm{~m}_{\mathrm{DB}}^{\prime-} & =1-\left(1-\mathrm{m}_{\mathrm{DB} \leftarrow\{\mathrm{DB} .1, \mathrm{DB} .2, \mathrm{DB} .3\}}^{-}\right)\left(1-\mathrm{m}_{\mathrm{DB}}^{-}\right) / \mathrm{K}_{\mathrm{DB}},  \tag{23}\\
\mathrm{~m}_{\mathrm{DB}}^{\prime \Theta} & =\mathrm{m}_{\mathrm{DB} \leftarrow\{\mathrm{DB} .1, \mathrm{DB} .2, \mathrm{DB} .3\}}^{\Theta} \mathrm{m}_{\mathrm{DB}}^{\Theta} / \mathrm{K}_{\mathrm{DB}},  \tag{24}\\
\mathrm{~K}_{\mathrm{DB}} & =\left(1-\mathrm{m}_{\mathrm{DB} \leftarrow\{\mathrm{DB} .1, \mathrm{DB} .2, \mathrm{DB} .3\}}^{+}\right)\left(1-\mathrm{m}_{\mathrm{DB}}^{+}\right) \\
& +\left(1-\mathrm{m}_{\mathrm{DB} \leftarrow\{\mathrm{DB} .1, \mathrm{DB} .2, \mathrm{DB} .3\}}^{-}\right)\left(1-\mathrm{m}_{\mathrm{DB}}^{-}\right)  \tag{25}\\
& -\mathrm{m}_{\mathrm{DB} \leftarrow \mathrm{DB} .1, \mathrm{DB} .2, \mathrm{DB} .3\}}^{\Theta} \mathrm{m}_{\mathrm{DB}}^{\Theta},
\end{align*}
$$

Equations (18) through (25) determine the belief masses at all the variables at Level 2.

## Step 5: Propagation of Beliefs from Level 2 to Level 1

This step is the final step in the derivation of the audit risk formula at the balance sheet level. We again use [8] to propagate the beliefs from Level 2 to Level 1 and combine these beliefs with the beliefs obtained from the evidence directly bearing on the variable $B$ at Level 1 using [7]. The resulting belief masses are:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{B}}^{\prime+}=1-\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \text { all X2 }}^{+}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{+}\right) / \mathrm{K}_{\mathrm{B}},  \tag{26}\\
& \mathrm{~m}_{\mathrm{B}}^{\prime-}=1-\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \text { all X2 }}^{-}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{-}\right) / \mathrm{K}_{\mathrm{B}} \text {, }  \tag{27}\\
& \mathrm{m}_{\mathrm{B}}^{\prime \Theta}=\mathrm{m}_{\mathrm{B} \leftarrow \operatorname{all~} \mathrm{X} 2}^{\Theta} \mathrm{m}_{\mathrm{B}}^{\Theta} / \mathrm{K}_{\mathrm{B}} \text {, }  \tag{28}\\
& \mathrm{K}_{\mathrm{B}}=\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \text { all X2 }}^{+}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{+}\right)+\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \text { all X2 }}^{-}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{-}\right)  \tag{29}\\
& -m_{B \leftarrow a l l ~ X 4}^{\Theta} m_{B}^{\Theta} \text {, }
\end{align*}
$$

where $\mathrm{X} 2 \in\{\mathrm{Ai}, \mathrm{DB} ; \mathrm{i}=1,2,3, . . \mathrm{n}\}$.

## Audit Risk Formula at the Balance Sheet Level

From Equations (26) - (28), we obtain the following beliefs that overall the balance sheet is free from material misstatements, b , and has material misstatements, $\sim \mathrm{b}$ :

$$
\begin{align*}
& \operatorname{Bel}_{\text {total }}(\mathrm{b})=1-\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \text { all X2}}^{+}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{+}\right) / \mathrm{K}_{\mathrm{B}},  \tag{30}\\
& \operatorname{Bel}_{\text {total }}(\sim \mathrm{b})=1-\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \text { all X2} 2}^{-}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{-}\right) / \mathrm{K}_{\mathrm{B}} . \tag{31}
\end{align*}
$$

The plausibility that material misstatements are present at the overall level is given by

$$
\begin{equation*}
\mathrm{Pl}_{\text {total }}(\sim \mathrm{b})=\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \mathrm{all} \mathrm{X} 2}^{+}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{+}\right) / \mathrm{K}_{\mathrm{B}} \tag{32}
\end{equation*}
$$

Srivastava and Shafer [2] argued that the plausibility that material misstatements are present in the balance sheet defines the audit risk. Thus, based on their definition, the overall audit risk formula at the balance sheet level is given by (32).

$$
\begin{equation*}
\text { Audit Risk }=\left(1-\mathrm{m}_{\mathrm{B} \leftarrow \mathrm{all} \mathrm{X} 2}^{+}\right)\left(1-\mathrm{m}_{\mathrm{B}}^{+}\right) / \mathrm{K}_{\mathrm{B}} \tag{33}
\end{equation*}
$$

where $\mathrm{m}_{\mathrm{B} \leftarrow \text { all } \mathrm{X} 2}^{+}$is given in Table 2 and represents the belief masses propagated from the variables at Level 2 to variable B .

Equation (33) along with Equations (1)-(29) presents a general audit risk model which can be used by the auditor not only for planning purposes but also for the evaluation purpose. As mentioned earlier, during the planning phase since the auditor has no knowledge about the actual nature of the audit evidence, he/she assumes the nature to be positive as argued by Srivastava and Shafer. However, while conducting the audit, the auditor may encounter all kinds of evidence, positive, negative, and mixed. Thus, our approach provides an appropriate way to combine all the evidence encountered on the actual audit and hence helps assess the audit risk even at the completion of the audit.

If the assessed audit risk is more than what the auditor can accept, say $\mathrm{Pl}_{\text {Total }}(\sim \mathrm{b})>0.05$, the auditor has several options. For example, the auditor can gather more or different audit evidence. If the belief that overall financial statement is materially misstatement, i.e., $\mathrm{Bel}_{\text {Total }}(\sim \mathfrak{b})$, is
still significantly high, say 0.10 or higher, then the auditor can negotiate with the client and adjust the account balances in error and reassess the audit risk. If the reassessed audit risk is acceptable, then the auditor could provide an opinion that the financials are fairly stated. However, if the auditee is not willing to adjust the account balances in error then the auditor may issue an opinion that is either qualified, adverse or incorporate a disclaimer of opinion (see [9] for alternative types of audit opinions).

If, given additional audit evidence, the total belief, $\mathrm{Bel}_{\text {Total }}(\sim \mathrm{b})$, is so high that additional evidence is unlikely to decrease the audit risk, $\mathrm{Pl}_{\text {Total }}(\sim \mathrm{b})$, to an acceptable level, and if the client is not willing to adjust the accounts, then again the auditor could provide a qualified or adverse opinion depending on the severity of the indicated errors.

As expected, audit risk as defined in Equation (33) and assuming that all the items of evidence are positive, i.e., all
$\mathrm{m}^{-} \mathrm{s}=0$, assuming the disclosure requirements are ignored, and assuming the inter-relationships between the balance sheet accounts and the classes of transactions are ignored, reduces to Srivastava and Shafer audit risk formula at the balance sheet level. Because of the lack of space we do not derive the audit formulas for Levels 3, 4, and 5. However, one can derive these formulas by using [7] and [8].

Table 2
Belief masses propagated from higher level3 to lower levels $\mathrm{m}_{\mathrm{X} 4 \leftarrow \mathrm{all} \mathrm{X} 5}^{+}=\prod_{\mathrm{X} 5} \mathrm{~m}_{\mathrm{X} 5}^{+}$, where X5 $\varepsilon$ \{CAi.j.k.1, CAi.j.k.2,CAi.j.k.3,
CAi.j.k.4, CAi.j.k.5,CDCAi.j.k\} and X4 $\varepsilon$ \{CAi.j.1, CAi.j.2, ...
CAi.j.k $\}, \mathrm{k}$ depends on the number classes of transactions associated with each variable at Level 4.
$\mathrm{m}_{\mathrm{X} 3-\text { all } \mathrm{X} 4}^{+}=\prod_{\mathrm{x} 4} \mathrm{~m}_{\mathrm{x} 4}^{+}=\prod_{\mathrm{x} 4}^{+}\left[1-\left(1-\mathrm{m}_{\mathrm{x} 4-\text { all } \mathrm{x} 5}^{+}\right)\left(1-\mathrm{m}_{\mathrm{x} 4}^{+}\right) / \mathrm{K}_{\mathrm{X} 4}\right]$
where $\mathrm{K}_{\mathrm{x} 4}$ is defined in Equation (9)
$\mathrm{m}_{\mathrm{X} 2 \leftarrow \mathrm{all} \mathrm{X} 3}^{+}=\prod_{\mathrm{X} 3} \mathrm{~m}_{\mathrm{X} 3}^{++}=\prod_{\mathrm{X} 3}\left[1-\left(1-\mathrm{m}_{\mathrm{X} 3 \leftarrow \operatorname{all} \mathrm{X} 4}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 3}^{+}\right) / \mathrm{K}_{\mathrm{X} 3}\right]$
where $\mathrm{K}_{\mathrm{x} 3}$ is defined in Equation (13).
X3 $\varepsilon$ \{Ai.1, Ai.2, Ai.3, Ai.4, DAi\}
$\mathrm{m}_{\mathrm{B} \leftarrow \mathrm{all} \mathrm{X} 2}^{+}=\prod_{\mathrm{X} 2} \mathrm{~m}_{\mathrm{X} 2}^{\prime+}=\prod_{\mathrm{X} 2}\left[1-\left(1-\mathrm{m}_{\mathrm{X} 2 \leftarrow \text { all X3 }}^{+}\right)\left(1-\mathrm{m}_{\mathrm{X} 2}^{+}\right) / \mathrm{K}_{\mathrm{x} 2}\right]$
where $\mathrm{K}_{\mathrm{x} 2}$ is defined in Equation (17).
$\mathrm{X} 2 \varepsilon\{\mathrm{~A} 1, \mathrm{~A} 2, \ldots \mathrm{An} . \mathrm{DB}\}$.

## V. CONCLUSION

In this paper we have derived a general audit risk formula at the balance sheet level in terms of belief masses, i.e., mvalues, obtained from various audit evidence gathered by the auditor. In our derivation, we have considered the possibility of mixed items of evidence, that is both positive and negative items of evidence. Also, we have integrated current AICPA's auditing standards [3] [4] which separates management assertions into three levels: Balance Sheet Accounts; Classes of Transactions; and Presentation and Disclosure. In addition, we extend the evidential diagram from just three levels as
treated by Srivastava and Shafer to two additional levels: Classes of Transactions and the associated assertions and disclosure requirements. As noted, our audit risk formula is based on a more realistic setting where both positive evidence which confirms the management assertions) and negative which implies that such assertions are not valid. Thus, importantly, the new risk formulations can be used not only for planning purposes but also for evaluation purposes

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Figure 1: Evidential Diagram for the Audit Process. The circles with a symbol ' $\&$ ' represents the AND relationship between the variable on the left with the variables on its right. The various assertions are listed in Table 1.


[^0]:    ${ }^{1}$ Management assertions are the implicit assertions made by the management when they publish company's financial information to imply that the financial information presented in the report present fairly the financial position of the

[^1]:    company. The AICPA [4] has published these assertions as listed Table 1.

