On the interest of having different combination rules in the evidence theory framework

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Abstract—**In the framework of the theory of belief functions, several rules have been defined to combine the considered evidences, for instance the orthogonal sum of Shafer, the conjunctive rule defined by Smets, the hybrid rule proposed by Dubois and Prade and more recently the cautious rule defined by Denoeux. In this study, having shown that none of these rules is systematically better than the others, several combination rules are put in competition, and the partial decisions coming from each rule are combined. This latter combination corresponds to a second level of fusion having logical sources given by the different combination rules. Its interest relative to decision making from only one combination rule is clearly stated on the considered examples.**

Keywords: Evidence theory, combination rules, logical sources, two-level fusion.

I. INTRODUCTION

The framework of the evidence theory [1] is now widely used for data fusion since it allows modelling the absence of information when sources only deal with a part of the discernment space (e.g. [2]). Using evidential framework, different combination rules are possible: in particular, the initial Shafer combination that redistributes the conflict on every focal assumptions, the conjunctive combination defined by Smets that keeps the conflict on $\varnothing$ hypothesis, the Yager rule and Dubois and Prade one that redistribute the conflict on specific compound hypotheses. Then, having combined the sources, a decision should be taken. Once more, several decision rules are possible, depending in particular on the set of hypotheses between which the decision should be taken (typically only singleton hypotheses or also compound hypotheses). Here, we propose to take advantage of the numerous combination rules to construct a new decision rule in order to obtain a more robust final decision eventually in favour to compound hypotheses.

In Section 2, the considered combination rules are briefly presented, and we show that none of them gives better results than the others in a systematic and significant way. Section 3 discusses the interest of a second level of data fusion combining the results obtained considering different combination rules. Section 4 illustrates the proposed approach showing some result examples. Section 5 gathers our conclusions.

II. COMPARISON OF SOME COMBINATION RULES

Let us focus on the problem of $|\Theta|$ singleton hypotheses (where $|.|$ denotes the cardinal of a set), and n logic sources, noted S_i ($i\in\{1,\dots,n\}$) In the framework of the evidence theory, the handled assumptions are the Θ subsets, i.e. $\{H: \varnothing\subseteq H\subseteq\Theta\}$. We denote m_i ($i\in\{1,\dots,n\}$) the mass function (or basic belief assignment bba) associated to source S_i. These are combined using one of the following combination rules.

The 'orthogonal sum' was firstly defined by Shafer [1] to combine evidences. Then, Zadeh [3] pointed out its main drawback as masking the conflict between sources and Smets [4] proposes to remove the normalization in the orthogonal sum. Now, the so-called 'conjunctive rule' drawback is that the conflict increases with the number of sources combined. Thus, some rules, namely the 'Yager one' [5] and the 'hybrid Dubois and Prade rule' [6], have proposed to redistribute the conflict on compound hypotheses.The mathematical expression of these rules for two sources S_i and S_j (bbas m_i and m_j) and a discernment space noted 2^Θ, with Θ the union of all singleton hypotheses and $\varnothing$ the empty set is as follows:

Conjunctive rule [4]:

$$\forall H\in 2^\Theta,\ m(H)=\sum_{X,Y\in 2^\Theta / X\cap Y=H} m_i(X).m_j(Y)=m_{i\cap j}(H). \quad (1)$$

Yager rule [5]:

$$\begin{cases} \forall H\in 2^\Theta-\{\varnothing,\Theta\}, m(H)=\sum\limits_{\substack{X,Y\in 2^\Theta/\\ X\cap Y=H}} m_i(X).m_j(Y),\\ m(\varnothing)=0,\\ m(\Theta)=m_i(\Theta).m_j(\Theta)+\sum\limits_{\substack{X,Y\in 2^\Theta/\\ X\cap Y=\varnothing}} m_i(X).m_j(Y). \end{cases} \quad (2)$$

Hybrid rule [6]:

$$\begin{cases} \forall H\in 2^\Theta-\{\varnothing\}, m(H)=\sum\limits_{X,Y\in 2^\Theta/X\cap Y=H} m_i(X).m_j(Y)+\\ \qquad\qquad \sum\limits_{X,Y\in 2^\Theta/\begin{cases}X\cap Y=\varnothing\\ X\cup Y=H\end{cases}} m_i(X).m_j(Y),\\ m(\varnothing)=0. \end{cases} \quad (3)$$

Conversely to orthogonal sum and conjunctive rule, the Yager rule and the hybrid combination rule are not associative. Indeed, let us consider three sources S_i and the associated bbas m_i, $i\in\{1,2,3\}$, and two hypotheses A and B, then the term $m_1(A)m_2(A)m_3(B)$ will contribute to $A\cup B$ mass if S_1 and S_2 are firstly combined (and then S_3) and to A mass if S_1 and S_3 –resp. S_2 and S_3– are firstly combined (and then S_2 –resp. S_1). According to the hybrid rule principle, as long as two sources are not conflicting, both are reliable, but when they are conflicting, only one is reliable. In this latter case, the right hypothesis belongs to the union of the conflicting hypotheses, and a third source is necessary to raise the ambiguity. Now, considering the three sources at once and X, Y, $Z\in 2^\Theta$, one can either assign the mass product $m_1(X)m_2(Y)m_3(Z)$ to $X\cup Y\cup Z$ mass when $X\cap Y\cap Z=\varnothing$ (the generalization to more than three sources is immediate), or follow the principle of the 'third' source raising the ambiguity, similarly to a '3-source voting' rule [2].

Lets note $|W|$ the cardinal of a set W, $\{X,Y,\ldots,Z\}$ a n-uplet of hypotheses of the space of discernment 2^Θ, and $\{W\in 2^\Theta : H\subseteq W\}$ the set the hypotheses (singletons and compound hypotheses) including the singleton hypothesis H ($|H|=1$). Then, $|\{X,Y,\ldots,Z\}\cap\{W\in 2^\Theta : H\subseteq W\}|$ is the cardinal of the subset of the n-uplet $\{X,Y,\ldots,Z\}$ including H hypothesis. In the following, it is noted $|\{X,Y,\ldots,Z\}\cap H|$. Finally, we define a 'dominant' hypothesis H for the n-uplet $\{X,Y,\ldots,Z\}$ as a singleton or compound hypothesis such that:

$$H = \bigcup_{\begin{cases} H_i \text{ singleton hypothesis} \\ |\{X,Y,\ldots,Z\}\cap H_i| = \max\limits_j\left(\left|\{X,Y,\ldots,Z\}\cap H_j\right|\right) \end{cases}} H_i \quad . \tag{4}$$

$$\forall H\in 2^\Theta, m(H) = m_{1\cap 2\cap\ldots\cap n}(H) + \sum_{X,Y,\ldots,Z\in 2^\Theta / \begin{cases} X\cap Y\cap\ldots\cap Z=\varnothing \\ H \text{ dominant for } \{X,Y,\ldots,Z\} \end{cases}} m_1(X)m_2(Y)\ldots m_n(Z). \tag{5}$$

In the following, we call this combination rule the 'n-voting' rule. It is a generalization of the '3-source voting' rule presented in [2] in the case of three sources and three hypotheses, allowing mass expression specification. Note also that in the case where n equals 2, the 'n-voting' rule reduces to the hybrid rule [6].

Property: according to the q-ordering or the pl-ordering, the bba resulting of the 'n-voting' rule, noted $\text{bba}_{\text{n-voting}}$, is more committed (MC) than the bbas resulting from the Yager rule, noted $\text{bba}_{\text{Yager}}$, and than the bba resulting from the Dubois and Prade rule, noted $\text{bba}_{\text{hybrid}}$.

Proof: H dominant for $\{X,Y,\ldots,Z\} \Rightarrow H\subseteq X\cup Y\cup\ldots\cup Z$, thus from a $\text{bba}_{\text{hybrid}}$ to a $\text{bba}_{\text{n-voting}}$, mass has been redistributed only from compound hypotheses X to included hypotheses $H\subseteq X$, thus $\forall H\subseteq\Theta$, $q_{\text{n-voting}}(H)\geq q_{\text{hybrid}}(H)$, thus $\text{bba}_{\text{n-voting}}$ is q-MC than $\text{bba}_{\text{hybrid}}$. In the same way, we show that $\text{bba}_{\text{n-voting}}$ is pl-MC than $\text{bba}_{\text{hybrid}}$, and since $\text{bba}_{\text{hybrid}}$ is q&pl-MC than $\text{bba}_{\text{Yager}}$, so does $\text{bba}_{\text{n-voting}}$ by transitivity. ■

Previous rules assume that the sources are independent. When it is not the case, the cautious rules that are idempotent and able to combine non-distinct evidences are preferable. Recently, [7] proposes a cautious rule based on the canonical decomposition of bbas that allows estimating the source correlation in every non-dogmatic case:

$$m = \bigcap\nolimits_{X\subset\Theta} X^{\min\left(w_i(X), wj(X)\right)} \tag{6}$$

$$\forall X\subset\Theta, \begin{cases} w_i(X)\in(0,+\infty), \\ \ln\left(w_i(X)\right) = -\sum\limits_{Y\supseteq X}(-1)^{|Y|-|X|}\ln q(Y). \end{cases} \tag{7}$$

Figure 1 shows, for three different kinds of data, the repartition (in percentage) between Right Decisions (RD) and False Decisions (FD) obtained using rule Rj, among the False Decisions obtained using rule Ri. In every case, three sources and three hypotheses are considered, after combination the decision is taken according to the maximum of pignistic probability [4], and the number of data samples for one simulation (1, 2 or 3) is 100000. For simulations 1 and 2, the sources are independent and the mean class distance ranges from 2 to 1, the class variance being constant and equal to 1. In simulation 3, the class parameters are the same as for simulation 1, but the sources 1 and 2 are partially correlated, and the class standard deviation of source 3 is 0.1.

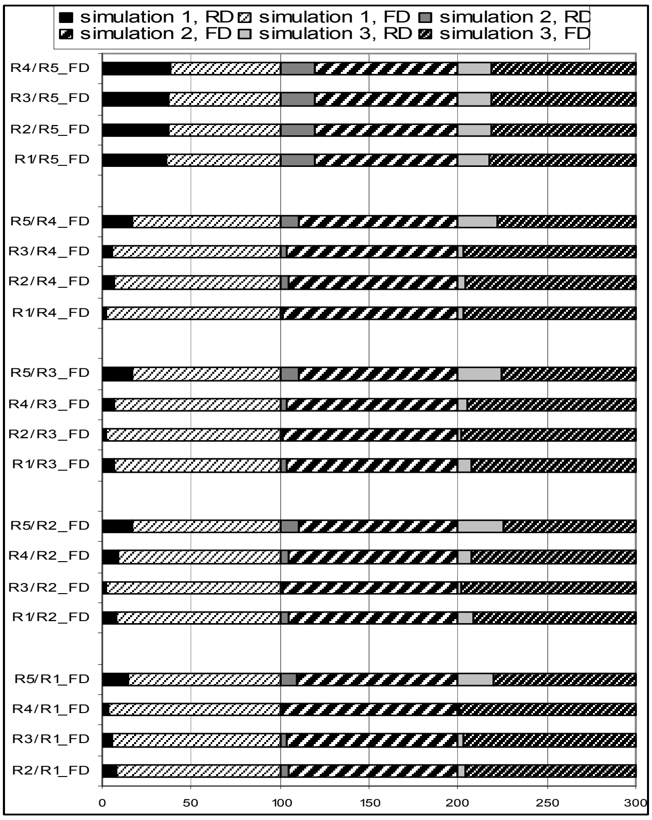


Figure 1: % of Right Decisions (RD) and False Decisions (FD) using combination Rj, among the False Decisions using rule Ri, noted Rj/Ri_FD, with $(i,j)\in[1,5]\times[1,5]$, $i\neq j$, R1: conjunctive rule, R2: Yager's rule, R3: hybrid rule, R4: n-voting rule, R5: cautious rule. In simulations 1-2, sources are independent conversely to simulation 3 case.

First of all, we check that in no case the repartition between RD and FD is 0%-100% (nor 100%-0%), that means that whatever the considered combination rule, there are some errors done that are not done considering another combination rule, in other words there is no universal combination rule. Second, we note that some combination rules are more redundant than others: for instance, the cautious rule is much less correlated to the other rules than the others between themselves. We also note that this 'correlation' between combination rules does not clearly depend on the simulation whereas the rule performance does. In particular, the cautious rule is efficient only in the simulation 3 case (more precisely, the rates in % of RD are as follows: simulation 1: R1: 78.3, R2: 78.1, R3: 78.3, R4: 78.5, R5: 71.3; simulation 2: R1: 57.8, R2: 57.6, R3: 57.8, R4: 58.0, R5: 52.8; simulation 3: R1: 70.3, R2: 68.8, R3: 69.0, R4: 69.8, R5: 71.2). Indeed, the right hypothesis has a higher probability to be designed by several sources, and in the absence of actual correlation between sources, it is detected as such. Now, using the cautious rule close beliefs (e.g. designing a same hypothesis) would be considered as non-independent and considered only once, preventing the accumulation of evidences on the right hypothesis. This is why some authors have proposed a variant of the cautious rule, namely the cautious adaptive rule [8], that is able to take into account the actual source correlation through a parameter. In the following, we will try to take advantage of the complementarity of the combination rules to improve the performance of fusion.

In conclusion, some general evidence properties, such as the independence or the level of conflict, should be considered in the choice of the combination rule. Therefore some authors (e.g. [9], [10]) have proposed a hierarchical approach with different combination rules used for different source clusters and fusion level. Besides, we also see that the exact performance of the different combination rules relative to each others remains unpredictable. Therefore, we consider them as as many sources having their own imprecision and uncertainty.

III. Second level of fusion

A. Principle

In this study, we propose a two-level fusion scheme as shown on Figure 2. The first-level sources are classical data sources, for instance measurements done by different sensors, or outputs of different data processing algorithms. At the first level, these sources are combined using the different evidential combination rules and produce as many outputs of the fusion first-level. These first-level outputs are the inputs of the fusion second-level, which are thus combined to obtain the second-level output.

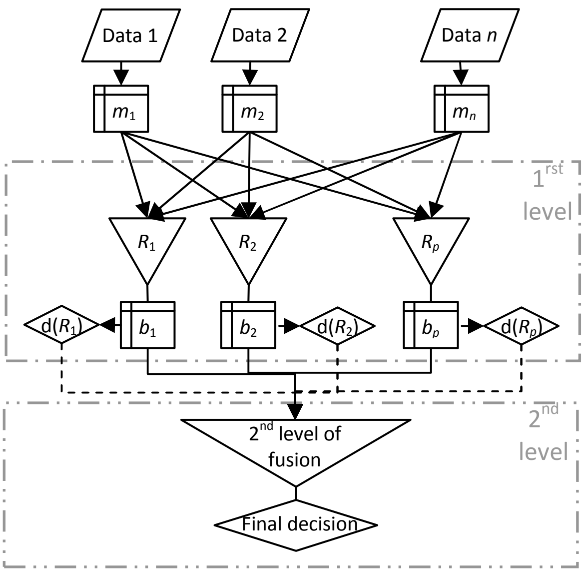


Figure 2: Scheme of the two-level fusion, in the case of n initial data sources and p combination rules; partial decisions using combination rules R_i are obtained as $d(R_i)$.

Introducing a second level in fusion, one important point is the formalism to use for this level. In data fusion, combination rules are generally classified between rules that produce more committed information, and rules that produce less committed information. For instance, the Smets conjunctive rule [4] and the Denoeux cautious rule [7] belong to the first kind of combination rules, whose generic name is 'conjunctive'. Examples of the second kind of rules are the classical disjunctive rule, obtained replacing $\cap$ by $\cup$ in Equation (1), or the bold disjunctive rule [7]. In the Yager rule, the conflict is reallocated to Θ, thus $\forall H \subseteq \Theta$, $pl_Y(H)=pl_\cap(H)+m_\cap(\emptyset)$, and $q_Y(H)=q_\cap(H)+m_\cap(\emptyset)$, where $pl()$ and $q()$ are the plausibility and the commonality functions respectively, and the subscript refers to the used combination rule, Yager's or conjunctive. Thus, the Yager rule produces a bba less committed than the conjunctive one, but in the general case nothing can be said concerning the ordering relatively to the initial bbas. The same occurs for the hybrid rule result: more committed than the Yager one and less committed than the conjunctive one, but either less or more committed than the initial bbas.

Now the basic idea of the two-level of fusion is as follows. At first level, there may be some so-called 'erroneous' strongly noised measurements that we aim at filtering by comparison with other data measurements. For this, we used one of the considered combination rules: conjunctive, Yagers', hybrid, *n*-voting, or cautious. Indeed, when the 'erroneous' measurements are in minority number and when they are considered at the beginning of the data fusion process in the case of non-associative rules, previous rules provide a bba where the impact of the 'erroneous' data sources has been smoothed. Then, the second level of fusion aim at taking advantage of the dispersion of first data fusion results to confirm or to refute the decision that could have been taken after the data fusion first-level. In other words, we consider that each first-level data fusion process is a data source that can be combined with other similar data sources to make the final decision more robust.

Now, these second-level sources can be considered as much more reliable than initial data sources, due to the data fusion first-level that filters the 'erroneous' measurements as explained. Thus, in most cases, they are not conflicting, and conflict mainly indicates that the bbas do not highlight one hypothesis but several ones, and that a precautionary principle could be to not decide between highlighted hypotheses. In such perspective, there are two key ideas of this second-level fusion: first one is 'rather be imprecise than wrong', and second one is 'conflict is an indicator of 'wrong' '.

Finally, as illustrated on Figure 2, the second level of fusion can be performed either considering the bbas b_i computed using the combination rule R_i at fusion first-level, or considering the partial decisions $d(R_i)$ associated to the b_i. In the following, we present two examples of fusion at second-level, the first one from $d(R_i)$, and the second one from b_i.

B. The consensus rule

In previous section, we define two key ideas for the second-level fusion using words 'imprecise' and 'conflict' that are now defined in the case of partial decisions. Here, the word 'imprecise' is defined equivalently in terms of cardinal of the decided hypothesis or commitment of the associated bba. Indeed, if we associate a Simple Support Belief (SSB) m_i to decision H_i such that $m_i(H_i)=1-a_i$, $m_i(\Theta)=a_i$, then the ordering between decisions and the ordering between these SSB bbas are consistent. Then, by extension of notation we said a decision in favour of an hypothesis H_1 is more committed, or more precise, than a decision in favour of an hypothesis H_2 if H_1 is strictly included in H_2 ($H_1 \subset H_2$, thus $|H_1|<|H_2|$). The association between decided hypothesis and SSB also allows defining the conflict: two hypotheses are conflicting if the mass of the empty set is non-null after conjunctive combination of their SSB, what boils down to the fact that their intersection is empty.

Here, we define the so-called 'consensus' rule that decides for the more committed hypothesis (singleton or compound), noted $\widetilde{H}$, that is not conflicting with the partial decisions $\widetilde{H}\lfloor m_{R_i} \rfloor$ obtained considering the bbas derived using combination rules R_i,. It writes:

$$\widetilde{H} = \bigcup_{\text{combination rule } R_i, i\in[1,5]} \widetilde{H}\lfloor m_{R_i} \rfloor \ . \tag{8}$$

We emphasize that the result does not depend on the number of sources 'voting' for a hypothesis, this is all the more important that the second-level sources are combination rules that may be not independent.

Let us also stress that defining such a decision rule revives the interest of having different combination rules at our disposal. In [11], R. Haenni criticised the interest of defining other combination rules than the orthogonal sum. Now, in the absence of an ideal solution (remind that even the orthogonal sum has been criticised [3]), considering several solutions is the very principle of fusion: each solution is assimilated to a data source and it is assumed that, in average at least, the decision taken considering all data sources is more optimal than any decision derived from individual sources.

C. The compromise rule

First, note that the principle of the so-called 'consensus' rule is similar to either the hybrid combination rule –in case of conflict between the beliefs in different hypotheses, the beliefs are reported on the compound hypothesis– or the disjunctive rule. Indeed, the consensus rule can also be obtained from one of these two combination rules provided that (i) associating to each $\widetilde{H}\lfloor m_{R_i} \rfloor$ a SSB such that a_i<<1; (ii) decision can occur also in favour of compound hypotheses, for instance as it will be defined latter; and (iii) in the combination, all the SSB are considered at once (only necessary for the hybrid rule).

In a similar way that the n-voting rule was defined to obtain a more committed (but not conflicting) result than those provided by the hybrid rule, we now investigate the possibility to have a decision both non-conflicting with and more committed than the consensus one.

Several ways to define a compromise rule were investigated. The first one was to try to remain in the evidential framework, and to apply one evidential combination rule: disjunctive rule, bold (disjunctive) rule, hybrid rule… In the following we only present some results obtained with the disjunctive rule. As explained latter a critical point is the decision rule to apply to the obtained bba.

The second way was to define an ad-hoc decision rule considering the different bbas. As tested ad-hoc decision rules work as follows: for each singleton hypothesis H chosen by at least one of the partial decisions (i.e. included in the consensus decision), they compute an index I(H) that will have to be maximized. For instance I(H) may be the maximum of pignistic probability among the partial bbas having decided H, or it can be the ratio between the maximum of pignistic probability and the second maximum of pignistic probability (still among the partial decisions), or the logarithm of the previous ratio… Then, the ad-hoc decision includes the singleton hypothesis that maximizes I(H) and all the singleton hypotheses that are closer to the ad-hoc decision than a threshold t:

$$\widetilde{H} = \bigcup_{\begin{cases} X\in\Omega, |X|=1, \\ \forall Y\subseteq H-X, I(Y)\geq I(X) \\ |I(H-X)|-|I(X)|<t \end{cases}} X \tag{9}$$

In our case, the index of a compound hypothesis is the minimum of the indices of the included singleton hypotheses.

Note that in the case of the disjunctive second-level rule, this decision criterion was also applied simply defining I(X)=BetP(X) for X singleton hypothesis. However results will show the lack of robustness applying such a decision criterion to a single bba.

IV. Examples of Results

In this section, we estimate the performance of the proposed approach on simulated data.

A. Data simulation and performance criterion

In all our simulations, we consider three (initial) sources and three singleton hypotheses in {A, B, C}. Every distribution is assumed to be Gaussian with standard deviation equal to 1 and class distance varying between 0.5 and 3.

Three kinds of mass allocation were considered. In the two first cases, for each data sample, the conditional probabilities to the classes (singleton hypotheses) are respectively either allocated to the corresponding singleton mass or distributed between corresponding singleton and the consonant compound hypotheses but Θ, before global discounting. Thus, on Figure 3, the mass functions cover the two lines when the global discounting coefficient a varies. The third mass allocation is as follows: for each class A, a consonant mass function is constructed with focal elements assumption including A, and then all these mass functions are combined according to the orthogonal sum.

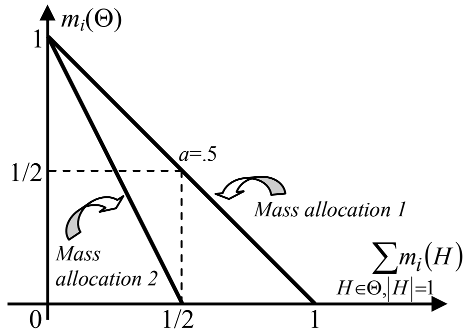


Figure 3: Representation of two ways to allocate the initial mass from class features and data sample.

Finally, in the three mass allocation cases, a random Gaussian centred noise is added on mass functions before re-normalization of these latter.

Performance is computed through the following score value:

$$score = \frac{100}{N_{test}} \times \sum_{k=1}^{N_{test}} c\left(\tilde{H}(k), \hat{H}(k)\right), \tag{10}$$

where N_{test} is the number of data random samples for one simulation, namely 100000, $\tilde{H}(k)$ and $\hat{H}(k)$ are respectively the estimated hypothesis (in {A, B, A∪B, C, A∪C, B∪C, A∪B∪C}) and the true hypothesis (in {A, B, C}) for sample k, and $c(.,.)$ is a cost function. In our case,

$$c\left(\tilde{H}, \hat{H}\right) = \begin{cases} \dfrac{1}{\left|\tilde{H}(k)\right|} & \text{if } \hat{H} \subseteq \tilde{H}, \\ -1 & \text{if } \hat{H} \cap \tilde{H} = \varnothing. \end{cases} \tag{11}$$

Thus the score may vary between -100 and 100, and the higher the score value, the more performing the evaluated algorithm. From score definition, we are able to evaluate not only decision in favour to a singleton hypothesis but also in favour to compound hypothesis. More precisely, a hypothesis is said 'correct' when it is not conflicting with the true hypothesis (it includes it from § III.B. definition of conflict between hypotheses and the fact the true hypothesis belongs to the singleton set), and its contribution to the score (to be maximized) is as important as it is precise (still as defined in § III.B), and it is said 'wrong' when it is conflicting with the true hypothesis (no possible belief transfer) and its contribution to score is negative (or null).

B. *Results*

Figure 4 upper shows the score versus the distance between class centres and for mass allocation case one, for different combination rules. The dashed curves indicate the results after the first level of fusion (using maximum of pignistic probability as decision criterion) and the continuous curves show the results obtained by the two-level fusion, either using the consensus rule or one of the compromise: ad hoc compromise or disjunctive one. Obviously the score increases with class centre distance. On the lower Figure 4, the difference of score values with the conjunctive rule result score are plotted. Positive (resp. negative) values correspond to an improvement (resp. decline) relative to the performance of the conjunctive combination rule.

First we note that the fusion first-level results are very close in term of score but the cautious rule. Indeed, as already said, the cautious rule prevents the accumulation of evidences that are close and so filters the evidence noise much less that other rules.

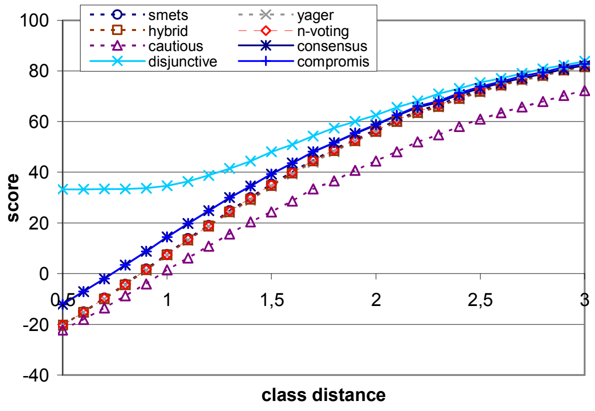
Second, concerning the compromise rules versus the consensus one, here we only show the best result (i.e. inducing the highest score) varying the threshold parameter. Thus, the compromise rule score can only be greater than or equal to those of the consensus rule (since for threshold value sufficiently large the compromise rule is equivalent to the consensus rule). On lower Figure 4, we see that for class distance values sufficiently large (namely 1.8 in the plotted case), the ad hoc compromise provides slightly better scores than the consensus rule. This confirms the presence of some supplementary information in the bbas at the output of the fusion first-level, information that is not used considering only the (first-level) decisions as does the consensus rule.

Third, concerning the second-level disjunctive rule, from Figure 4, results seem amazingly good: for very low class distance, the rule has the wisdom to not decide between hypotheses achieving a score of 33, and as the class distance increases, it becomes more committed taking right decisions so that it remains above all other score curves. However, as it will be illustrated by the next Figure, a strong drawback of this rule is its lack of robustness versus mass allocation process.

Figure 5 shows the score values obtained using the two-level fusions varying the initial mass repartition through the discounting coefficient, a. We clearly see that the very high performance of the disjunctive rule is due to the 'good' fitting of a: for a values higher than 0.3, it favours too much compound hypotheses (for a=.8, it always decides for Θ!), and for a values lower than 0.3, it is close to the other 2-level fusion results). Conversely, we note the very stable performance of the two other 2-level fusions, namely ad-hoc compromise and consensus.

Here we do not present the results obtained considering the other mass allocation processes, since they are essentially the same ones: the fusion second-level results have higher scores than the first-level ones, the compromise rules can achieve even better score than the consensus one, however the disjunctive compromise is very instable.

Note that a part of better performance of the 2-level fusion relative to the 1-level one is due to the score definition and the fact that maximum pignistic probability forces decision on singleton hypotheses. Now, the definition of a decision rule able to decide also in favour of compound hypotheses remains an open question as shown by the results obtained with the 2-level disjunctive compromise.



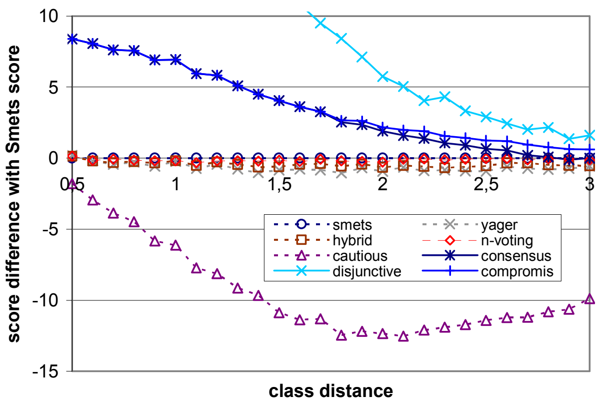


Figure 4: Score (upper) or score difference with conjunctive result score (lower) vs distance between classes, for different combination rules, namely Smets, Yager, Dubois and Prade (hybrid), 'n-voting' and Denoeux (cautious) combination rules for the fusion first-level, and consensus, ad hoc compromise ('compromis') or disjunctive rule for the fusion second-level; case of mass allocation 1 and discounting coefficient *a*=.3.

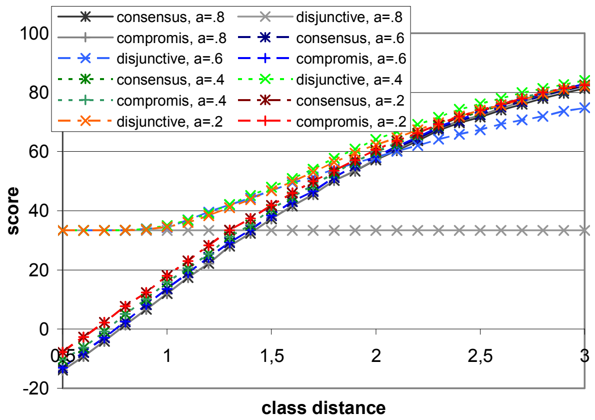


Figure 5: Score versus distance between classes, using different 2-level fusions, namely consensus, ad hoc compromise ('compromis') or disjunctive compromise, and for different discounting of initial bbas (cf. Figure 3).

V. Conclusion

Here we investigate the presence of information in the variability of the results produced by the different combination rules proposed in the evidence theory. By the way, we also a new combination rule is proposed called '*n*-voting' rule that allows to redistribute the conflict assuming a minor number of sources may be erroneous. Essentially, we propose to put in competition the partial bbas coming from each combination rule, namely Smets conjunctive one, Yager one, Dubois and Prade hybrid rule, the proposed '*n*-voting' rule, and the Denoeux cautious rule. Either some partial decisions, derived from each of these bbas, are combined according to the so-called 'consensus' decision rule —since partial decisions may be not independent, the 'consensus' rule simply decides in favour of the more committed hypothesis that raised the conflict between partial decisions, namely the union of these latter—; or these bbas are processed (combined or compared) and a rough decision rule able to decide in favour of compound hypotheses is applied.

The definition of the processes of the second level of the fusion is still an open research field. However, even with the very rough tested processes, we already show that there is effectively some information to get from the analysis of the results provided by the different combination rules. Nowadays this information is used mainly to be more cautious and decide for a compound hypothesis rather than for a singleton hypothesis. Obviously, the degree of cautiousness is linked to the score definition: with a cost for an error equal to 0, one has nothing to loss to bet for a singleton hypothesis, this is why we set this cost to a negative value (which is also in agreement with numerous practical applications where an error has a non-null cost!). Thus, another perspective is the definition of a decision rule able to choose compound hypotheses. Then it will raise the question of the robustness of such a rule relatively to the two-level fusion approach.

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