Pitfalls for recursive iteration in set based fusion

Gavin Powell[1], Matthew Roberts[12], David Marshall[2]

[1]EADS, Innovation Works, Newport, Wales, UK
[2]Geometric Computing and Computer Vision, School of Computer Science, Cardiff University, Cardiff, Wales, UK
gavin.powell@eads.com, m.s.roberts@cs.cf.ac.uk, david.marshall@cs.cf.ac.uk

***Abstract*— Set based methods, such as Dempster Shafer Theory and the Transferable Belief model, are widely used in fusion applications. Many fusion applications are of an iterative or recursive nature, often fusing data with respect to an event over time. This iterative or recursive approach does not translate well into the traditional set based fusion methods as the system can converge and become unresponsive. Documentation, analysis nor acknowledgement of these issues is readily available. In this paper, we highlight the problems that can occur, discussing the reasons for these problems and look at a previously suggested solution. We use some suitable simple examples to aid in the description.**

Keywords:*DST (Dempster Shafer Theory), TBM (Transferable Belief Model), DSmT (Dezert-Smarandache Theory), conflict, temporal, iterative, recursive*

I. Introduction

Iterative and recursive processes happen regularly in our world. Repeatedly we find the need to monitor these processes and events. Often this monitoring task will take in more than one source of information. This is a typical description of a fusion task.

Traditional set based fusion methods (e.g. DST, TBM, DSmT) are commonly used to perform fusion tasks [i,ii,iii]. However if the problem is not formulated correctly then severe issues will arise that can lead to erroneous results or incorrect conclusions that are drawn from them.

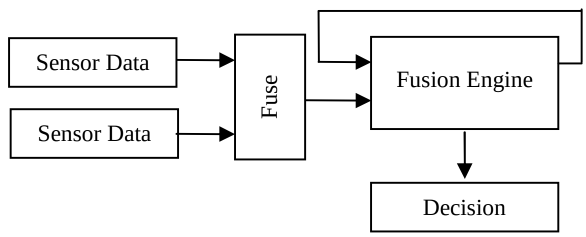

Figure 1. The Recursive Fusion Process

The main purpose of this paper is to highlight the issues that can arise from making inappropriate decisions to fuse data using set based methods in a recursive and iterative manner. One possible solution to this was proposed previously by the authors [iv]. A method of convergence protection was developed while tackling other fusion based problems. The issue of convergence was noted and an attempt to counteract it devised. The analysis of the act of convergence was not thoroughly investigated, nor an adequate solution found.

II. Paper Outline

In Section III we revisit the basics of DST and its various incarnations. We also look at how combination is generally carried out within DST and how this type of combination can skew results. In Section IV we look at the issues that can arise from iteratively and recursively using these rules for combining information. Highlighting where care needs to be taken when using DST based methods. In Section V we conclude

III. Set Theoretic Approaches

Early work by both A.P.Dempster [v] and Glenn Shafer [vi] later became known as Dempster Shafer Theory (DST). DST is a generalisation of Bayesian Theory and it states

1. beliefs are created from subjective probabilities
2. information is fused using Dempster's rule of combination

The basis of the work in DST has been taken and extended by other parties [vii,viii]. This basic framework has evolved into various mutations, but they share the same core idea. One such framework is the Transferable Belief Model (TBM) [vii] and we shall base the rest of the discussion on this to highlight the empty set issues, where appropriate we will compare and contrast to other approaches.

A. Basic Elements of the Transferable Belief Model

The TBM is a set theoretic approach, such as discussed previously. The TBM splits the set theory into two stages. Firstly the *credal* level where your beliefs are entertained and quantified by belief functions. Secondly, the *pignistic* level where those beliefs are used to make decisions and are quantified by probability functions.

For illustration purposes lets consider a simple weather example: Let us assume that all of the possible weather types that we are going to account for in our world are *wind*, *snow* and *rain*. Therefore, the set of all the possible types of weather we know about is given by

$\Omega = \{w = wind, r = rain, s = snow\}$ and all possible combinations

$$\Theta = [\{w\},\{r\},\{s\},\{w,r\},\{w,s\},\{r,s\},\{w,r,s\},\varnothing] \quad (1)$$

To this we apply a basic belief assignment (bba)

$$m : 2^{\Omega} \rightarrow [0,1] \text{ with } \sum_{A \subseteq \Omega} m(A) = 1 \quad (2)$$

where *m(A)* is the basic belief mass (bbm) given to A.

This is a method of assigning masses to each of the subsets of Ω to signify our belief, or that there is evidence showing that the truth is somewhere within that set, not necessarily equal to but, within that set. Each of these sets is in fact a hypothesis. Any set that has a *bbm > 0* is called a *focal* set, and any focal set that only has one member is called a *singleton* set. The more members within a focal set the more the *ignorance*, or uncertainty about which single state is true.

One of the possible subsets of Θ is the empty set $\varnothing$. Values assigned to it carry various meanings which depend on whether we are working within an *open* or *closed* world. A *closed* world is one where we assume that we have modeled every possible outcome and that we have full and complete knowledge about our problem space. Every possible outcome that we are considering is included in **Ω**. Values given to the empty set in this case are due to imprecision and thus it is just redistributed after any data combination. In contrast an *open* world assumes that we do not know everything about the world we are modeling and we accept that there is a possibility of something else existing. If the world is open then we have accounted for everything we know about in **Ω**, but also accept that there *may* be something else in the world that we don't yet know about (for example, sun), and this is given by the empty set, $\varnothing$.

Traditionally DST approaches take place with a closed world [ix] and TBM an open world [x].

B. *Combining Data*

If we have more than one piece of data from the same source over time, multiple sources at the same time, or even multiple sources over time, then it is normal that we will want to fuse or combine all of this data. This combination will enable us to make a more informed decision, using all available information, as opposed to just looking at a single piece of evidence. An example of such a task would be predicting the weather based on previous weather data and current forecasts from multiple sources. We want to recursively combine, or fuse, this weather forecast data, before we make an educated decision on the most likely weather for tomorrow, based on all the information that has been made available to us.

Each piece of evidence (weather forecast data) will give us a bba over Θ such that bbm's are applied to the various sets within Θ. This will show where we think the truth is (the forecast for tomorrow). If this evidence is very certain then the singleton sets within Θ will get more mass, if we are uncertain then the set **Ω** will get more mass. Coincidently the set that shows complete ignorance is **Ω**. A completely naive state would apply all of the bbm to **Ω**. To fuse data we must combine these bba's to create a new bba. There are a multitude of methods to accomplish this [xi]. The original work on the DST used Dempster's (conjunctive) rule of combination shown in Equation 3

$$m_{1\otimes 2}(A) = \frac{\sum_{B\cap C} m_1(B)m_2(C)}{1-k} \quad (3)$$

$$\text{where } k = \sum_{B\cap C=\varnothing} m_1(B)m_2(C)$$

It can be seen that *k* is in fact a normalisation factor that redistributes any mass assigned to the empty set after combination. If we are to remove the normalisation we get Dempster's un-normalised rule of combination as used in the TBM [vii] and shown in Equation 4.

$$m_{1\otimes 2}(A) = \sum_{B\cap C} m_1(B)m_2(C) \quad (4)$$

For example, let us envisage that we have two weather forecasting experts that have given us two bba's where **Ω**={snow,rain,wind).

Use of Equation 4 on these bba's will give the results shown in Table 1, where B is the horizontal grey and C is the vertical grey and the remaining cells show $B \cap C$ and the mass of $m_1 \times m_2$. The summation part of Equation 4 will add up the masses given in these cells that have the same set elements.

Table 1 shows us that a lot of mass $m_{1\otimes 2}$ is going to be assigned to the empty set. In fact 42% of the operations give rise to values being placed in the empty set.

Figure 2 shows the proportion of the values, after combination, are given to the empty set. The instance of 3 singletons, as with Table 1, gives a ratio of 0.42. This shows that 42% of all combination operations result in an empty set, this is also shown in Table 1. The hypothetical case of 0 singletons would clearly give a dominance ratio of 0 as the empty set, $\varnothing$, would be the only element in Θ. It should be noted that not every combination will apportion this amount to the empty set. Generally not all of the sets will have a belief assigned to them prior to combination, and also the beliefs are not even distributed across the sets. This means that two pieces of information could be combined with nothing going toward the empty set, if they are in complete agreement.

Table 1.
SET INTERSECTION AS USED IN DEMPSTERS RULE OF COMBINATION

	m_1(snow)=0.1	m_1(rain)=0.1	m_1(wind)=0.2	m_1(snow, rain)=0.1	m_1(snow, wind)=0.2	m_1(rain, wind)=0.1	m_1(snow, rain, wind)=0.1	m_1(∅) =0.1
m_2(snow)=0.1	snow 0.01	∅ 0.01	∅ 0.02	snow 0.01	snow 0.02	∅ 0.01	snow 0.01	∅ 0.01
m_2(rain)=0.2	∅ 0.02	rain 0.02	∅ 0.04	rain 0.02	∅ 0.04	rain 0.02	rain 0.02	∅ 0.02
m_2(wind)=0.1	∅ 0.01	∅ 0.01	wind 0.02	∅ 0.01	wind 0.02	wind 0.01	wind 0.01	∅ 0.01
m_2(snow, rain)=0.2	snow 0.02	rain 0.02	∅ 0.02	snow, rain 0.02	Snow 0.04	rain 0.02	snow, rain 0.02	∅ 0.02
m_2(snow, wind)=0.1	snow 0.01	∅ 0.01	wind 0.02	snow 0.01	snow, wind 0.02	wind 0.01	snow, wind 0.01	∅ 0.01
m_2(rain, wind)=0.1	∅ 0.01	rain 0.01	wind 0.02	rain 0.01	wind 0.02	rain, wind 0.01	rain, wind 0.01	∅ 0.01
m_2(snow, rain, wind)=0.1	snow 0.01	rain 0.01	wind 0.02	snow, rain 0.01	snow, wind 0.02	rain, wind 0.01	snow, rain, wind 0.01	∅ 0.01
m_2(∅)=0.1	∅ 0.01	∅ 0.01	∅ 0.02	∅ 0.01	∅ 0.02	∅ 0.01	∅ 0.01	∅ 0.01

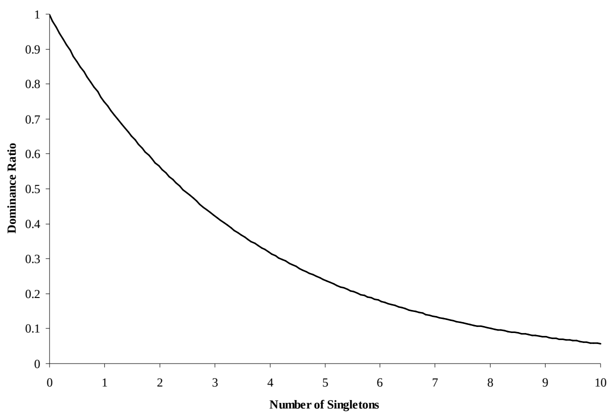

Figure 2. Empty set dominance ratio after combination

The point we are trying to raise is that using this type of combination rule *can* send a lot of the mass toward the empty set, as has been mentioned previously [xii]

Other combination rules exist [xi] that are able to redistribute a proportion of the empty set, after each iteration, thus preventing convergence on the empty set. One of the best performing of these being PCR5 [viii], but possibly not the most efficient.

If we are not combining in a recursive manner then this should not be an issue. Many applications require recursive fusion. Using the open world and Dempster's conjunctive rule of combination together in a recursive nature can cause problems.

We use a simple example to display some of the issues that can arise when dealing with iterative application of Dempster's rule of combination. We are again looking at fusing weather forecast information over time. The forecast data places mass (beliefs) on two possible weather types thus $\Omega = \{snow, rain\}$.

IV. ITERATIVE FUSION

A. Set Convergence

We will iteratively and recursively fuse using $m(snow) = 0.9$ and $m(rain) = 0.1$. This is signifying a simple fusion example where the input is very sure that *snow* is the correct forecast for tomorrow's weather, and stays with this hypothesis over the time period.

Figure 3 shows the result of using an open world approach and the conjunctive rule of combination. It can be seen that even though the incoming data is constant in its beliefs, the empty set still begins to dominate. This can be explained by the way the two pieces of information are combined as shown in Table 1. Any conflict or uncertainty, even if small as in this case, will accumulate over time and cause the empty set to converge rendering the system unresponsive [viii]. This clearly is not a good outcome.

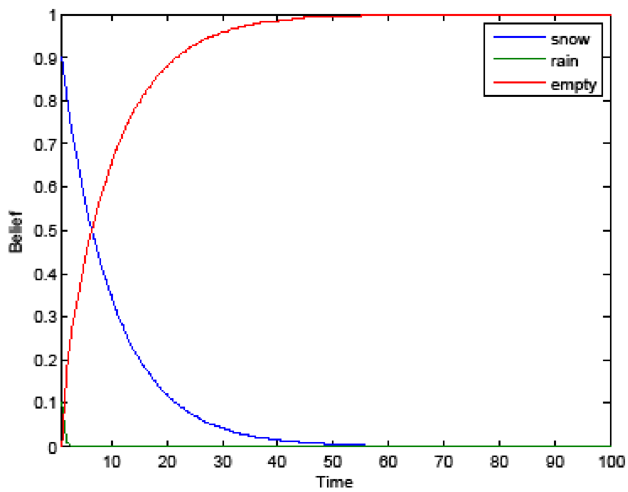

Figure 3. Empty Set dominance open world

If we take the same input as for Figure 3 but use a closed world where the conjunctive rule of combination, and normalisation occur, then we achieve the results as shown in Figure 4.

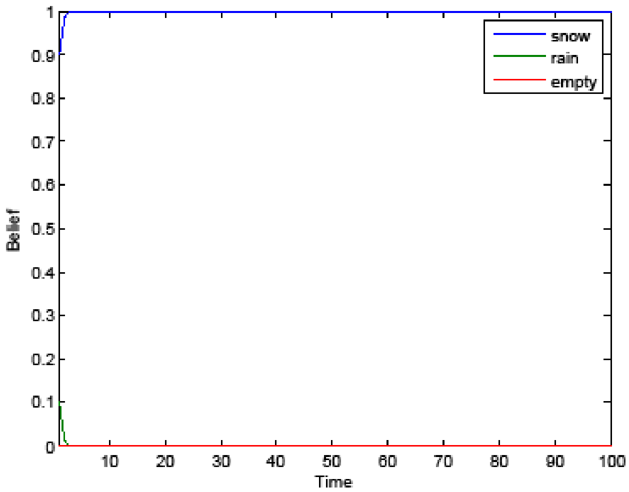

Figure 4. Normalised closed world

When normalising we quickly converge to whichever set is getting more mass, or belief. Initially it may appear that this is an acceptable result, showing high certainty for the set with most belief. This may not be the case where we have uncertainty in our input.

If we have incoming information that is uncertain as to the correct hypothesis such that *m{snow} = 0.55* and *m{rain} = 0.45* then you expect the system to maintain a level of uncertainty. But it will in fact quickly converge to the set (from the incoming data) with the greatest mass, as shown in Figure 5. If you were to make decisions based on the weather forecast, and the system reports its forecast is *snow* with 99% accuracy then the system would be giving you a false sense of security about the confidence of the overall weather type forecast.

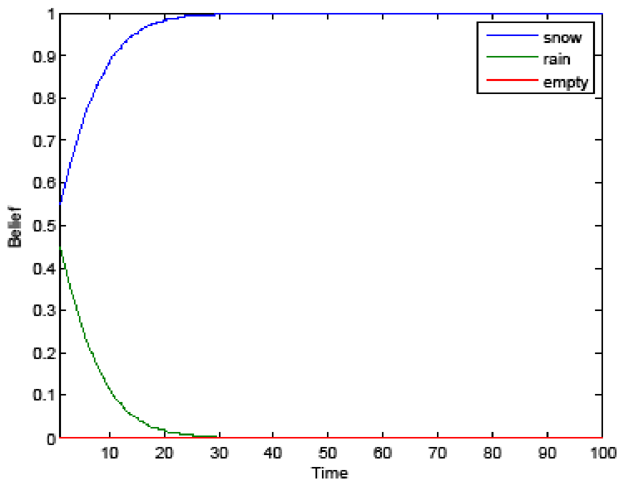

Figure 5. Normalised Closed world similar hypothesis

B. Dynamic Ability

Often with data fusion scenarios the sensing devices may change their outlook. This can occur as new information is received, viewpoints or resolutions change. Because of this it is important that a system remains dynamic, and flexible to changes in incoming information. Open and closed world approaches using the conjunctive rule of combination can quickly, and unnecessarily, converge. Once this convergence has taken place it is difficult to make the system recover.

If for frames 1-25 we use *m{snow} = 0.9* and *m{rain} = 0.1* and then for frames 26-100 we use *m{snow} = 0.1* and *m{rain} = 0.9* then we can simulate a very harsh change in the information that we are receiving from the sensors. Figure 6 shows that change with an open world and the conjunctive rule of combination. The change increases the conflict in the system and makes the convergence and dominance of the empty set more rapid. This small change in belief, and the ineffective manner that the TBM would deal with it, really highlights how ineffective this approach is for recursive and iterative procedures.

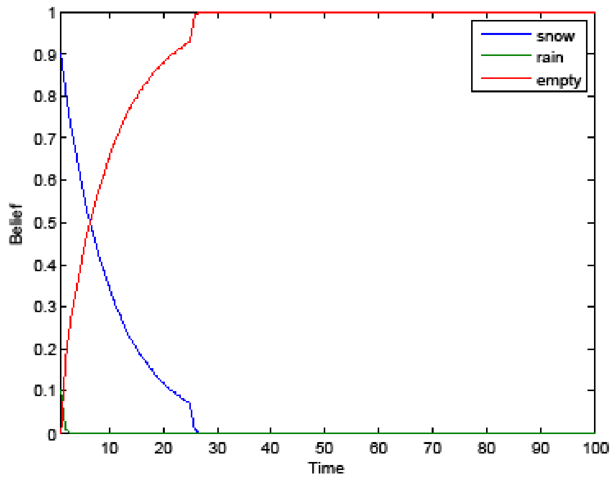

Figure 6. Open world target type change

Figure 7 shows how a closed world conjunctive combination reacts to the same inputs as used in Figure 6. Even though the change occurs at 25 frames it can be seen that it takes another 25 frames for the system to react. When you add some extra uncertainty and real world noise to the system then this delay would increase quite possibly to the extent that a change in outlook from the sensors is actually ignored. This is a very dangerous situation to be in when critical decisions need to be made.

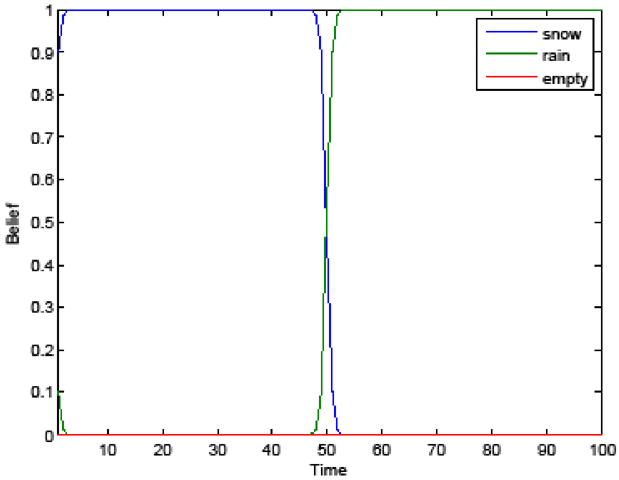

Figure 7. Closed world with changing beliefs

C. *Convergence Protection*

Previous work by the authors [iv] noted that convergence could be an issue in set based approaches. A simple measure to overcome this and provide convergence protection within the closed world was suggested. Figure 8 shows the results of applying the same data as used in Figure 7, but with convergence protection turned on as well. Convergence protection limits the amount of mass that can be assigned to any singleton set, after combination has taken place. Any mass that is in a singleton set and is over this limit is redistributed to the ignorant set Ω. This prevents the system from entering the state where it becomes unresponsive. You can then 'fine tune' the threshold of how dynamic the system needs to remain.

The results in Figure 8 show how the system is now able to adjust and adapt quickly to new incoming information.

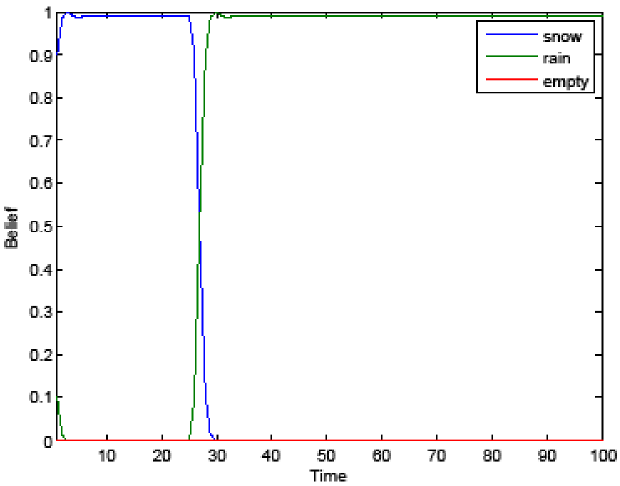

Figure 8. Closed world changing beliefs with convergence protection

D. *Application to Random Data Set*

To highlight the issues further we will contrast with the simplistic data sets used in Section IV by testing on a randomly generated set of beliefs. This random set is shown in Figure 9 and will test both set convergence and the systems dynamic ability to cope with change.

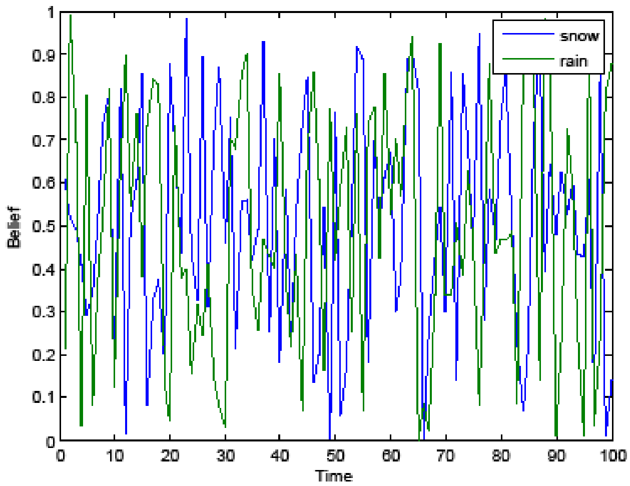

Figure 9. Random Input Beliefs

Figure 10 Shows how inadequate the open world, as used in the TBM, is when we recursively fuse data. The conflict being presented by the random data set quickly ensures that the empty set dominates.

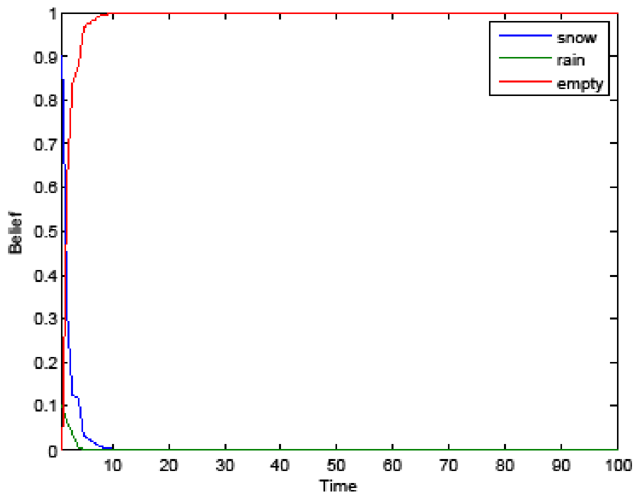

Figure 10. Open World Combination

Figure 11 and 12 show the effects of the convergence protection. Figure 11 shows that even when strong information is received, that is contrary to the current belief, the system can be slow to respond, to the extent that information is practically ignored. Figure 12 shows that with convergence protection the system can remain dynamic and respond well to input data.

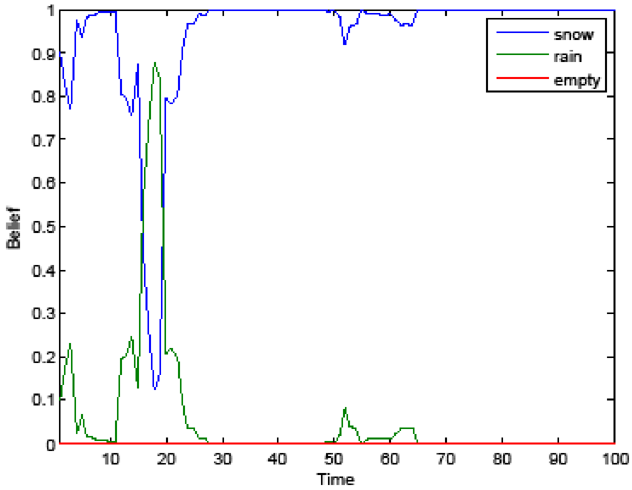

Figure 11. Closed World Combination

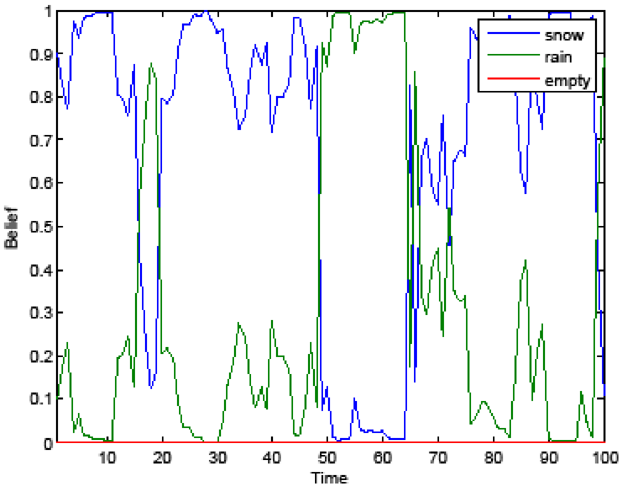

Figure 12. Closed World Combination and Convergence Protection

V. CONCLUSION

We have shown how iterative processes can present problematic results when we use set based fusion techniques. Initially it can seem as if the system is working well and providing a solid classification. Unfortunately this is often untrue and easily overlooked.

It can be seen that retarding the mass assigned to singleton sets and adding some uncertainty back into the system can aid in the process, making it much more flexible and dynamic to new information. But this in itself also causes problems as it can eradicate any 'memory' that the system has. If we have been receiving information regarding a certain event for some time then surely we should have some memory of this, thus making any new abrupt change to be viewed as noise. Effectively the system should know when to accept change and when to reject it. This is an avenue of further research for the authors.

Set based fusion and classification systems need to have a more inherent flexibility and intelligent approach to how they combine information and make classifications based on that. The authors feel that far too often DST approaches are used due to their simplicity, but are applied poorly, without an adequate understanding of the modeling required, nor of the interpretation of the results.

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