Characterization and propagation of uncertainties in complex socio-technical system risk analyses

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Abstract — **Risk analyses are often specific studies in various sectors (technical, human, organizational, environmental). However, facing the increasingly complex character of current industrial systems, it is important to deal with all sectors in a same risk model. This has led to develop a model that enables so-called "integrated" analyses to be performed. The studies carried out have demonstrated the difficulty in configuring these components accurately and/or completely. This observation leads us to ask questions related on how taking into account these uncertainties, modeling them and propagating them in this integrated model. The purpose of this paper is to apply elements of evidence theory to an industrial case in order to draw perspectives and to identify axes of future research.**

Keywords: Integrated Risk Analysis, Bayesian networks, uncertainties, evidence theory, evidential networks

I. Introduction

A. General context

At the present time, industrial systems are becoming more and more complex due to their increasing number of components and their interactions (example of instrumentation and control connecting components and software) and due to the recognition of employees and organizations interacting with technical systems. This is why the Integrated Risks Analysis (IRA) methodology has been developed by EDF in partnership with CRAN and INERIS for the analysis of so-called complex socio-technical systems [1] which comprise technical, environmental, human and organizational components. It is important to deal with all of these components in order to take their interactions into account in particular those of the various risks resulting from these components.

For instance, maintenance or operating actions are applied to a technical system but they are carried out by a team. Such operations are submitted to organization, regulatory conditions and physical environment context which influence both the system performance and the involved employees.

B. Orientation

The main goal of this IRA method is to be able to prioritize these different types of risks in order to better orientate reduction systems (called *barriers*) and to assist decision-making.

The estimation of these different risks is a critical phase in this risk analysis process. Indeed, it combines estimations which may be very precise and well quantified for technical (reliability of components) and environmental (statistics on hazards) risks, with qualitative estimations for the human and organizational aspects. Therefore several levels and types of uncertainties must be propagated in the model. Indeed, the considered industrial systems involving these different kinds of uncertainties cover many issues (safety, environment, economic, political, etc.) and different types of risks.

The Integrated Risk Analysis model developed in [1] is based on a **representation of the risks model under the Bayesian networks (BN) framework in order to deal with data issued from the experimental feedback and data provided by experts' judgment and inter-connected variables**. Consequently, the developments concerning the consideration of uncertainties should extend and support this formalism. For these reasons, in order to characterize the uncertainties associated with several types of estimations and to find a relevant model propagating them in the same representation, we investigate Evidential Networks (EN) model as proposed in [2] that appears as a solution to take into account different types of uncertainties and to integrate them in the Bayesian risks models based on the previous industrial studies.

This paper **aims at implementing the evidence theory on elementary patterns developed for IRA using BN. The final objective is the uncertainty treatment in the complex social and technical systems of industrial interest (many hundreds of inter-connected variables)**.

For this purpose, section II is dedicated to presenting the IRA method and a brief introduction of the industrial case. The needs for handling every kind of uncertainties are pointed out. section III concerns the modeling goals based on evidential networks to handle uncertainties. In the last sections, the most important elementary pattern of IRA models is tested and conclusions are provided.

II. INTEGRATED RISK ANALYSIS METHOD

A. *Principle of Integrated Risk Analysis (IRA)*

This method aims to take into account the complexity and interdisciplinarity of technical systems that are subject to maintenance and operating actions (carried out by employees involving in their organization) and the regulatory and physical environmental contexts. This method includes many goals and they must guarantee the various issues like safety, availability and maintaining the system during its lifetime.

The main challenges of this approach are to develop methods and tools for analyzing risks for systems subjected to correlated hazards and having correlated influences on the above defined issues. This is done in the aim of prioritizing different types of risks, helping the choice of barriers to reduce these risks and to contribute to their better control. It also contributes to the risk communication and to the choice of reduction or mitigation prevention barriers.

B. Conceptual framework of an integrated approach

The Integrated Risk Analysis process is based on a conceptual framework (Figure 1) enabling the various previously mentioned components to be taken into account and their interactions to be modeled [5].

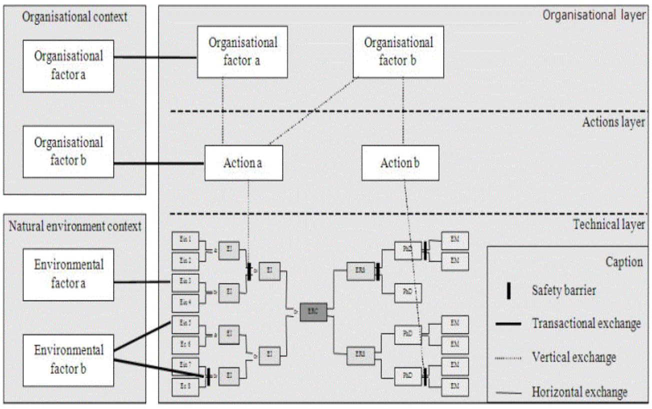

Figure 1. Conceptual framework.

Within this conceptual scheme, resulting from the Paté-Cornell and Murphy representation [3], the model covers three levels: technical, human and organizational, connected together by influence relationships related to the environmental and organizational contexts. The different levels of this model are characterised as follows.

1. Technical level: the system is represented by a bow tie [4], comprising a fault tree and an event tree, enabling the simulation of various scenarios that may arise (risk reduction and mitigation barriers are taken into account at this level)

2. Human level: it represents human actions (for example, maintenance or operating action). The behavior of the working team is therefore modeled using different *indicators* which enable the effectiveness of these actions to be assessed (for example: training, delegation, experience, etc.).

3. Organizational level: it represents the organization governing the above mentioned human actions using different indicators (called *POFs* for *pathogenic organizational factors*). This involves characterizing the different pathogenic elements affecting the technical system, here via human actions (for example: weakness shortcomings in the organization culture of safety, failure in daily safety management, production pressures, etc.) [5] and [6].

These different human indicators and POFs are distributed according to three phases characterising a human action: preparation, execution and closure. Various interactions among indicators and POFs are described in [1].

C. Modeling IRA type studies

Last step of an IRA involves representing models in order to carry out simulations and diagnoses. Due to the works led since several years by Leger *et al* [1] to model complex socio-technical systems, the available model is a BN.

This modeling formalism is justified by the ability of BN to represent multiple-attributes correlated variables, to combine data from expert's judgements and feedback. It also performs quick simulations or diagnoses. Thus the BN model is well adapted to the representation of complex technical systems. Nevertheless, the consideration of human and organizational factors raises the problem due to the uncertainty and imprecision in our studies. Therefore, the objective is not to lead a new modeling but to extend those existing and to enrich them by integrating it the handling of epistemic uncertainty.

III. MODELLING BY EVIDENTIAL NETWORKS

A. Notion of uncertainty

In our IRA models, two types of uncertainties are distinguished, which are:

- *aleatory uncertainty:* caused by the natural variability of a physical phenomenon (for example, behavior during failure differs from one component to another in the same series),
- *epistemic uncertainty:* caused by the imprecise or incomplete character of the information (for example, specific failure modes for a component are unknown).

B. Goals to be targeted

Risk studies are often tainted with these two types of uncertainty. In the technical field, it is possible to have access to data that enable probability distributions to be constructed. However, the characterization of human actions and their organizational context within this integrated vision of risks implies to study alternatives representation of uncertainty. **The classic probability framework does not allow taking into account epistemic uncertainty.** It is thus necessary to switch to another uncertainty modeling method (ensuring the previously defined industrial goals).

Indeed, the construction of probability distributions would involve having access to information that would be impossible to obtain for human and organizational factors. For these reasons, the analyst develops a quantification based on verbatim in which the interviewed participants described the action, its complexity, the level of training of participants, their experience, and the aids used, etc. It is therefore possible to define the **minimum and maximum bounds of the risks** associated to the considered action.

Indeed, as interval valued probabilities cannot be easily considered in BN, a different model was chosen, EN. They consist in a graphical representation of knowledge like BN but integrate belief notions. Ben Yaghlane and Mellouli [7] propose EN based on Dempster combination rules (DCR) and the generalized Bayesian theorem (GBT) [8] whereas Simon and Weber propose EN based on Dempster-Shafer structures and Bayesian inference extended to belief masses. They propose a formalism applied to performance, reliability and utility analyses that characterizes uncertainty with minimum and maximum values by considering DS structures and which allows to deal with a lot of variables.

The theory of belief functions is one solution for the treatment of uncertainties. Indeed, we can quote other methods such as: imprecise probabilities [9], possibility theory [10], upper and lower probabilities [11], fuzzy sets [12], etc. Nevertheless, EN is very interesting in our application, because this modeling method is very close to the BN modeling method. EN allows using the structure of the BN model used in the IRA studies and extends it to model the epistemic uncertainty. In the following, we study the usefulness and feasibility of EN modeling method for Integrated Risk Analysis (IRA) of a real industrial case.

C. *Evidence theory*

The evidence theory was developed by Dempster during his work on upper and lower probability bounds in 1967 [13] and then completed by Shafer with a more complete mathematical formalism in 1976 [14]. Generally seen as a generalization of the probability theory, it may be based on a modeling using probability intervals, and corresponds to a relaxation of the basic axioms of probabilistic theory. Consequently, new notions were defined to characterize the study of variables involved in risk models based on the theory of belief functions. For our studies, we use the following notations:

- State hypothesis *i* for variable *X*: H_i^X
- Focal set associated with variable *X*: A_j^X
- Belief mass of A_j^X : $m(A_j^X)$
- Belief of A_j^X : $Bel(A_j^X)$
- Plausibility of A_j^X : $Pls(A_j^X)$

D. *From Bayesian networks to evidential networks*

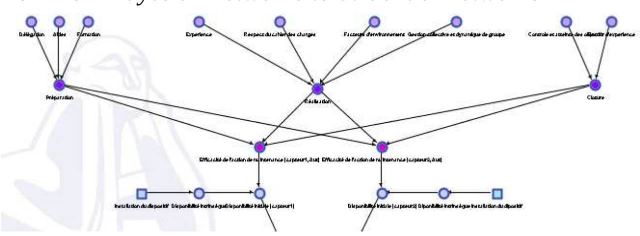

Figure 2. Example of IRA Bayesian elementary pattern.

Systems studies through the IRA method are often complex, i.e. with an important number of variables and dependence links between them. Figure 2 shows an elementary pattern of the IRA model (due to the confidentiality restrictions, the global model can not be presented in this paper) which is repeated several tens of time in risks model. Representing these systems to study them requires the use of a tool allowing modeling that; BN is a solution to build this model as presented in [1].

The use of combination between evidence theory and BN as a support for the propagation of uncertainties in the IRA model is explained, on the one hand, by the use of BN to IRA models and, on the other hand, by the necessity of using the concepts of evidence theory to a industrial large scale. Moreover, this theory allows taking epistemic uncertainty into account in the BN used for the IRA.

Tools developed at present such as IPP Toolbox (by EDF R&D and University of Duisburg-Essen [15]), TBMLAB (by Smets [16]) or EN [8], although effective, do not allow, at present, to be used on large-size industrial systems cases (with generally several hundreds of nodes connected between them by many dependence relations) as we can meet them in risks analysis in nuclear or transport domains for example. It is thus necessary to investigate another way of modeling allowing to take into account uncertainties in IRA studies.

EN as proposed by Simon and Weber [2] allows implementing the evidence theory to the industrial world by leaning on its bases and combining them with the properties of the Bayesian inference. They are similar to BN and allow a modeling of the dependences and the expression of relations between the various variables of the study on a common structure based on graphs. The difference between these two models is marked at the level of *semantic* (modalities and probabilities in BN and focal sets and masses in EN). The *inference* based on junction tree [17] can be used in EN as proposed in [2]. In the next section, we test them on the elementary pattern found in IRA models.

IV. IMPLEMENTATION IN A RISK MODEL

A. *Industrial context and characterization of uncertainty*

In IRA studies (generally applied to nuclear systems or high safety systems) the objective is to determine variables issues with the most possible precision that represent the studied installations (safety, availability and maintenance of the system during its lifetime). Due to the criticality of these installations, the results have to be the more close possible of the reality to guarantee accurate risks estimation and allowing the best decision-making. Two concepts related to IRA (section II.B.) are likely to cause uncertainty in the studies:

- *Influencing factors:* they characterize the influence that a variable of the network will have on its downstream variable. For example, the state of a human indicator characterizing the preparation of maintenance action influences its effectiveness.
- *Prior distributions:* they are defined on the basis of experimental feedbacks or even on expert opinions. They characterize the prior state of the variable. For example, the presence/degradation of an initial condition.

Concerning influencing factors (denoted α), they are assigned to the relationship between two nodes based on an expert elicitation grid (Figure 3) according to the impact of the upstream variable on its downstream variable. The value represents the lower limit of α. For instance, in the case of a *little impact* α = 0.7 and if there is *no impact* α = 0.9. It is difficult, for the expert, to decide between two consecutive values of α in the elicitation grid. Moreover the value in Figure 3 represents an interval as defined in the Figure 4.

Here appears the importance of the Dempster-Shafer theory, which helps to take into account this uncertainty and enables the influencing factor to be defined by means of intervals.

Parameters	α
No impact	0.9
Little impact	0.7
Impact	0.5
High impact	0.3
Total impact	0.1

Figure 3. Elicitation grid of the influencing factor α.

Parameters	α
No impact	[0.9 ;1]
Little impact	[0.7 ; 0.9[
Impact	[0.5 ; 0.7[
High impact	[0.3 ; 0.5[
Total impact	[0.1 ; 0.3[

Figure 4. Elicitation grid for the factor α using intervals.

The prior distribution is based on the observations made on already studied test-cases (Table I). However in some cases, the modality of the variable is not perfectly known (for instance if a component is out of order its failure state is unknown before diagnosis or if an indicator is not observed every modalities are possible). The uncertainty can be taken into account as proposed in Table II. These tables given for illustration are those used for defining indicators in the following simulations.

Table I.
PRIOR DISTRIBUTION WITHOUT IMPRECISION

Indicator	
{present}	{degraded}
0.65	0.35

Table II.
PRIOR DISTRIBUTION WITH IMPRECISION

Indicator		
{present}	{degraded}	{present,degraded}
0.6	0.3	0.1

Table III.
TABLE DEFINING THE BEL(EFFECTIVE) NODE

Preparation	Bel(effective)	
	Believe	Doubt
{effective}	1	0
{ineffective}	0	1
{effective,ineffective}	0	1

Table IV.
TABLE DEFINING THE PLS(EFFECTIVE) NODE

Preparation	Pls(effective)	
	Plausibility	Disbelief
{effective}	1	0
{ineffective}	0	1
{effective,ineffective}	1	0

These two configurations of uncertainty (on the influencing factors and on the prior distributions) are studied in a test case initially represented using BN. The BN is transformed to an EN to take into account the various uncertainties. Thus we use focal sets and masses propagation in the EN model used for IRA study.

B. Uncertainty propagation test case

To study the propagation of epistemic uncertainty within an EN, the simple case of V-structure is used allowing the study of various sources of uncertainty. In this test case, we use a recurrent elementary structure of the global IRA networks. It allows understanding easily the modeling method. Then, this reasoning can be applied to the whole model.

The model consists of two nodes characterizing two human indicators describing a phase for preparing maintenance or operating action, itself described by the variable *Preparation*. Two other nodes are added to these three nodes enabling the computation [2] of belief and plausibility measure of the effectiveness of the preparation phase to be characterized (Figure 5).

For each of the human indicators, two states are defined: {*present*} if the indicator complies fully with the previously defined criteria for characterizing it, {*degraded*} if the indicator does not comply with one or more of the criteria defining it. The impact of the human indicator will be more or less high according to the sensitivity of the variable *Preparation* to the human *Indicators* 1 and *Indicators* 2.

A *quantification method* [3] is used in order to compute the influence of the indicators on the *Preparation phase*. Influencing factors α_i are used to define the relation between the *Indicator i* and the *Preparation* phase. When several indicators are in state {*degraded*} the influence is given by the product of α_i of these indicators.

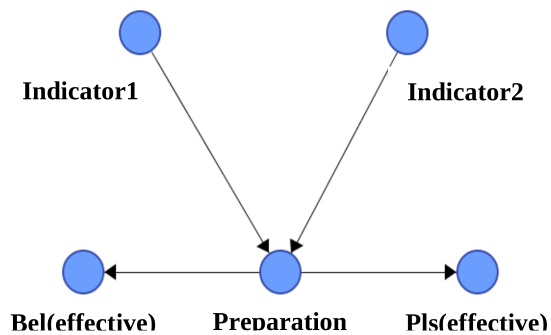

Figure 5. Model used for the tests.

This model is used for various uncertainty inclusion configurations: no uncertainty on the model and data (case 1), uncertainty on the influencing factors linking the human indicators and the phase preparation that they describe (case 2), uncertainty on the state of the human indicators (case 3), uncertainties on the influencing factors and the state of the human indicators (case 4).

For each simulation, nodes are defined describing the presence or degradation of human indicators with the same distributions (Tables I and II) and this in order to compare the various inferences on the network. The *conditional masses table* (CMT) associated with *preparation* node depends on the studied case. These CMTs (equivalent of *conditional probabilities tables* in BN) enable the distribution of the masses to be characterized according to the various *focal sets* defined in the study. The nodes dedicated to compute the belief and plausibility of preparation phase effectiveness are described using tables (Tables III and IV).

Case 1: no uncertainty on the model and data.

This case is the commonly used configuration and is a Bayesian case. The CMT related to the *Preparation* node (Table V) is therefore fairly easy to complete.

Table V.
CMT LINKED TO CASE 1

indicator1	indicator2	Preparation {effective}	{ineffective}	{effective,ineffective}
{present}	{present}	1	0	0
	{degraded}	0.7	0.3	0
{degraded}	{present}	0.5	0.5	0
	{degraded}	0.35	0.65	0

Table VI.
CMT LINKED TO CASE2

indicator1	indicator2	Preparation {effective}	{ineffective}	{effective,ineffective}
{present}	{present}	1	0	0
	{degraded}	0.7	0.1	0.2
{degraded}	{present}	0.5	0.3	0.2
	{degraded}	0.35	0.37	0.28

If both human indicators are present then there will be no impact on the effectiveness of the preparation of the action (the influencing factors α are therefore equal to 1) leading to 100% of effectiveness. If one or both indicators are degraded, it will reduce the effectiveness of the preparation.

As there is no epistemic uncertainty in this model, the prior distribution of *{effective,ineffective}* modality only comprises zero values.

Case 2: uncertainty on the influencing factors linking the human indicators and the phase of preparation.

This case only deals with influencing factors which are included in intervals. As both factors here are initially set at 0.5 and 0.7, intervals [0.5; 0.7] and [0.7; 0.9] are retained.

Changes in the previous case will occur when one or even both indicators are degraded. Indeed, the factors α rely within the intervals. Consequently, if *Indicator* 2 is degraded (its factor is included in the interval [0.7; 0.9]) we end up at worst with effectiveness reduced to 0.7 and ineffectiveness at 0.3 (α=0.7) and at best effectiveness at 0.9 and ineffectiveness at 0.1 (α=0.9). Thus, the 0.2 enabling passage from 0.7 to 0.9 of efficiency is situated in *{effective,ineffective}* modality, characterizing uncertainty. If both indicators are degraded, the sum of both influencing factors is calculated on the intervals. Therefore, in this second case, we will have Table VI.

Case 3: uncertainty on the state (present or degraded) of the indicator upstream of the phase that it describes (Table VII).

Case 4: uncertainties on the influencing factors and on the state of the indicators give the next table.

Table VII.
CMT LINKED TO CASE 3

indicator1	indicator2	Preparation {effective}	{ineffective}	{effective,ineffective}
{present}	{present}	1	0	0
	{degraded}	0.7	0.3	0
	{present,degraded}	0.7	0	0.3
{degraded}	{present}	0.5	0.5	0
	{degraded}	0.35	0.65	0
	{present,degraded}	0.35	0.5	0.15
{present,degraded}	{present}	0.5	0	0.5
	{degraded}	0.35	0.3	0.35
	{present,degraded}	0.35	0	0.65

Table VIII.
CMT LINKED TO CASE 4

indicator1	indicator2	Preparation {effective}	{ineffective}	{effective,ineffecti ve}
{present}	{present}	1	0	0
	{degraded}	0.7	0.1	0.2
	{present,degraded}	0.7	0	0.3
{degraded}	{present}	0.5	0.3	0.2
	{degraded}	0.35	0.37	0.28
	{present,degraded}	0.35	0.3	0.35
{present,degraded}	{present}	0.5	0	0.5
	{degraded}	0.35	0.3	0.35
	{present,degraded}	0.35	0	0.65

V. RESULTS ANALYSIS

A. *Inference and first results*

Once these different cases have been defined and modeled, simulations are carried out and provide with the following results presented in a summarized way. Various inferences on each of the four cases have been carried out under the previously defined conditions (i.e. a distribution with or without uncertainty and with defined CMT).

For case 1, the probability of effectiveness is 0.74. The results on Bel and Pls bounds can be explained because no uncertainty (neither variability nor epistemic) has been considered on the indicators and the influencing factors. Here the obtained probabilities are only considered as references for the following cases which deal with uncertainties. All of the results are gathered in Table IX and Figure 6.

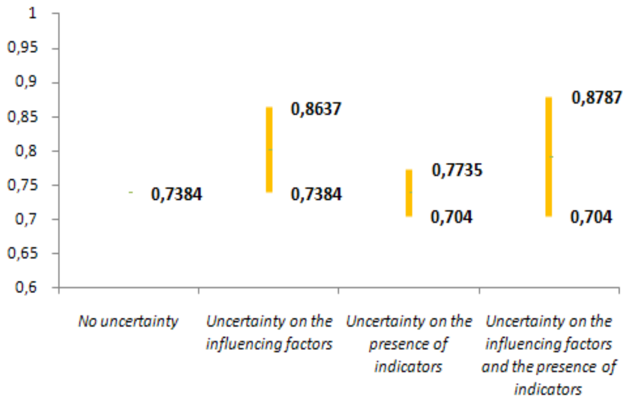

Figure 6. Intervals of effectiveness probability for the action related to each case.

Table IX.
VALUES OF THE BEL AND PLS BOUNDS OF CASE STUDIED

	Bel(effective)	Pls(effective)
Case 1 (no uncertainty)	0.7384	0.7384
Case 2 (uncertainty on α)	0.7384	0.8637
Case 3 (uncertainty on the indicators)	0.704	0.7735
Case 4 (uncertainty on α and the indicators)	0.704	0.8787

B. *Contribution of uncertainties*

Apart from its ability to propagate the uncertainty through the risk model, this exercise put forward its ability to prioritize the contributions of uncertainties on the model inputs to the results. Indeed, in the test case without uncertainty, the probability of effectiveness of the maintenance action is 0.74

For example, considering this value as the reference used for comparing the results including uncertainty, taking into account uncertainties on the influencing factors and the prior distributions on human indicators leads to a 0.17 range around this reference value with a spectrum centered above this value.

But the largest uncertainty affecting the effectiveness of the preparation phase is related to the uncertainty on the influencing factors α (smaller than the uncertainty on the prior distribution of indicators). This leads to give more emphasis to the influencing factor estimation phase than to the study of the presence/degradation of indicators.

However, the size of these intervals [Bel, Pls] mainly depends on the initially degree of uncertainty in these two configurations, i.e. on the influencing factors or on the prior distributions. Indeed, if the uncertainty on the indicators is higher, this would result in greater imprecision of the results.

As far the uncertainty on the influencing factors is concerned, it will always be approximately 0.2 (due to choice of the elicitation grid which makes it impossible to multiply the number of levels) whereas the presence/degradation of an indicator will always be more easily elicited by an expert and therefore it will be easier to reduce the related uncertainty (for example, by having better or broader experimental feedback).

Estimating influencing factors for this risk model will always be the most critical step of this risk assessment phase.

VI. CONCLUSION

A. Characterization of uncertainties in IRA thanks to the evidence theory

This implementation has demonstrated the feasibility of characterizing the variability and epistemic uncertainty and its propagation through large-scale EN (hundreds of nodes). These first results highlight the role that uncertainties play in the basic blocks of integrated risk models and consequently the need to take them into account in global models in order to increase the representativeness of our results and therefore to support a better decision-making. The lower value obtained for the effectiveness of the action when taking into account the uncertainty compared to the one obtained for the case without uncertainty indicates that the result with no uncertainty was optimistic. However, IRAs generally need to be conservative in order to ensure safety criteria.

Through this paper are able to put into practice the notion of the evidence theory on a real industrial complex case to model the uncertainties met during an IRA, in particular with the integration of human and organizational factors in risk analyses which are more often direct around the technical system. This modeling of uncertainties was possible due to a transformation of a large BN (used for IRA models) to a large EN by adapting semantics used for BN to that defined in the evidence theory and by using existing inference methods (junction trees). Moreover, these works were able to be realized on the basis of existing BN which models our system to take both aleatory and epistemic uncertainties into account in the same model.

This transition allows us the propagation of uncertainty at a lower cost as leaning on models and tools of treatment which already exist and because the stage of allocating a distribution function followed by one of random ranging (which moreover do not guarantee that ranging will be carried out all over the whole distribution including its tail when the number of calculations is not high enough) are not carried out.

B. Perspectives

Applied here to an elementary motive of a characteristic network of an IRA to study feasibility of the propagation of uncertainties in this type of analysis, it is henceforth necessary to develop these works on the whole network to study feasibility in a “large scale”.

It is also advisable to study the sensibility of the values in the table defining the influence factors, on one hand to assure the robustness of the works and on the other hand to refine this table and the associated intervals (intervals which are used in particular to the risks assessment by experts opinions).

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