A Dempster-Shafer Theoretic Evidence Updating Strategy for Non-Identical Frames of Discernment

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***Abstract*— How one may most effectively incorporate soft evidence into the fusion process has attracted considerable attention because soft evidence often provides the most critical information in battlefield and homeland security application domains. The Dempster-Shafer theoretic framework is a better candidate to capture the types of models and uncertain rules that are more typical of soft evidence. In this paper, we propose the *conditional update equation (CUE),* a Dempster-Shafer theoretic strategy for evidence updating that relies on the conditional approach for evidence fusion. The CUE can handle sources possessing non-identical scopes and provides a more reasonable solution when confronted with contradictory information, which constitute two major concerns related to incorporating soft evidence into the fusion process. We highlight several intuitively appealing properties that give further credence to the CUE.**

Keywords: Evidence updating, soft information, Dempster-Shafer theory, non-identical frames, conditional approach.

I. Introduction

Motivation: In asymmetric threat environments, the evidence possessing the highest value typically comes from *soft* sensors, such as COMINT (e.g., communication chatter, telephone records), HUMINT (e.g., informant and interrogation statements, domain expert inputs), and OSINT (e.g., open-source intelligence, such as, newspapers, Internet blogs, databases). Unfortunately, the absence of appropriate models for the error characteristics and imperfections associated with soft sources [1] makes it difficult to fuse soft evidence with the evidence generated from the more conventional physics-based *hard* sensors. This has created an enormous burden on intelligence officers who are forced to sift through and assess volumes of soft information for decision-making purposes. The question of how the more 'qualitative' information in soft evidence can be captured and fused with the more 'quantitative' information in hard evidence for increased automation of the decision-making process is attracting considerable attention from the evidence fusion community [2], [3].

Challenges: Bayesian probabilistic framework has difficulty in capturing the 'non-numerical' models that are more typical of soft evidence [1]. Probabilistic models' inability to preserve the material implications of propositional logic statements that are represented by uncertain rules [4], [5] constitutes a serious drawback that limits their utility in capturing soft evidence. On the other hand, models that are based on the Dempster-Shafer (DS) belief theoretic framework [6] can capture such uncertain rules while preserving the material implications of propositional logic statements that such rules represent, viz., reflexivity, transitivity, and contra-positivity [5].

DS theoretic fusion of soft evidence however must account for sources that may not have the same scope, or *frame of discernment (FoD)* in DS theoretic jargon. This is due to the fact that soft evidence is likely to be generated from a variety of sources having dissimilar FoDs. For example, the information contained in a public database of vehicles belonging to town residents, would have a much larger, but not completely disjoint, scope than the vehicles that had been recorded at a checkpoint. This constitutes a major drawback of the *Dempster's combination rule (DCR),* the de facto DS theoretic evidence fusion strategy. Deconditioning approaches address this limitation by artificially introducing ambiguities so that the FoDs are 'deconditioned' or 'expanded' to be identical [7], [8]. This requires one to work with an FoD of unnecessarily high cardinality, an unacceptable situation especially when large public databases may need to be consulted.

Fusion of hard and soft information can also involve evidence that is highly contradictory. For example, hard evidence can often be used to narrow down the time of occurrence of an event. On the other hand, regarding the same event, or perhaps regarding the underlying environment (e.g., the threat level), soft evidence spans a much wider time interval. Indeed, an event time interval provided by soft evidence might even precede, or even be disjoint with, the event time interval that hard evidence provides. The DCR tends to produce counter-intuitive results when it encounters contradictory evidence.

Contributions: The DS theoretic conditional approach in [9] takes a fundamentally different view: instead of requiring the FoDs to be 'expanded,' it enables a source to update its knowledge base by 'sifting' through the incoming evidence to take in only what it can discern or is interested in. The *conditional update equation (CUE)* for the identical FoDs case in [10] embraces this point of view. In this paper, we extend this CUE to accommodate non-identical FoDs.

This paper is organized as follows: Section II provides a review of essential DS theoretic notions; Section III proposes our new conditional update strategy that can accommodate non-identical FoDs; Section IV presents several of its interesting and intuitively very appealing properties; Section V contains an example; and, the concluding remarks appear in Section VI.

II. Preliminaries

A. Dempster-Shafer (DS) Theory

In DS theory, the total set of mutually exclusive and exhaustive propositions of interest is referred to as its *frame of discernment (FoD)* $\Theta = \{\theta_1, \ldots, \theta_n\}$ [6]. A singleton proposition θ_i represents the lowest level of discernible information. Elements in the power set of Θ, 2^Θ, form all the propositions of interest. We use $A \setminus B$ to denote all singletons in A that are not in B; $\overline{A}$ denotes $\Theta \setminus A$.

Definition 1: Consider the FoD Θ and $A \subseteq \Theta$.

(i) The mapping $m_\Theta(\bullet) : 2^\Theta \mapsto [0,1]$ is a *basic belief assignment (BBA)* or *mass assignment* if $m_\Theta(\emptyset)=0$ and $\sum_{A\subseteq\Theta} m_\Theta(A)=1$. The BBA is said to be *vacuous* if the only proposition receiving a non-zero mass is Θ.

(ii) The *belief* of A is $Bl_\Theta(A)=\sum_{B\subseteq A} m_\Theta(B)$.

(iii) The *plausibility* of A is $Pl_\Theta(A)=1-Bl_\Theta(\overline{A})$. ■

DS theory models the notion of *ignorance* by allowing the mass assigned to a composite proposition to move into its constituent singletons. A proposition that possesses non-zero mass is a *focal element.* The set of focal elements is the *core* $\mathfrak{F}_\Theta$*;* the triple $\{\Theta, \mathfrak{F}_\Theta, m_\Theta(\bullet)\}$ is the corresponding *body of evidence (BoE).* While $m_\Theta(A)$ measures the support assigned to proposition A *only,* the belief represents the total support that can move into A without any ambiguity; $Pl_\Theta(A)$ represents the extent to which one finds A plausible. When focal elements are constituted of singletons only, the BBA, belief and plausibility all reduce to a probability assignment.

Definition 2 (Dempster's Combination Rule (DCR)): The DCR-fused BoE $\mathcal{E}\equiv\mathcal{E}_1\oplus\mathcal{E}_2=\{\Theta, \mathfrak{F}_\Theta, m_\Theta(\bullet)\}$ generated from the BoEs $\mathcal{E}_i=\{\Theta_i, \mathfrak{F}_{\Theta_i}, m_{\Theta_i}(\bullet)\}$, $i=1,2$, when $\Theta\equiv\Theta_1=\Theta_2$, is

$$m_\Theta(A) = \sum_{C\cap D=A} m_{\Theta_1}(C)\, m_{\Theta_2}(D)/(1-K),\ \forall A \subseteq \Theta,$$

whenever $K = \sum_{C\cap D=\emptyset} m_{\Theta_1}(C)\, m_{\Theta_2}(D) \neq 1$. ■

Note that $K \in [0,1]$ is an indication of the conflict between the evidence provided by the BoEs. Hence, K is referred to as the *conflict* between the BoEs being fused. The DCR's difficulties in fusing conflicting BoEs are well documented. The requirement that the two FoDs being fused be identical constitutes another drawback associated with the DCR.

To fuse evidence generated from non-identical FoDs Θ_1 and Θ_2 (so that $\Theta_1 \neq \Theta_2$ and $\Theta_1 \cap \Theta_2 \neq \emptyset$), one can simply ignore the differences in the FoDs by having each source allocate zero mass to propositions that are not within its own FoD and continue applying DCR. So, this approach assumes that each source can discern $\Theta_1 \cup \Theta_2$ and ignores the fact that some propositions are not within its scope of expertise. The counter-intuitive conclusions this approach may generate are well documented [7]. In the deconditioning approaches, each source would artificially introduce ambiguities into its evidence so that its own FoD is 'expanded' to $\Theta_1 \cup \Theta_2$.

B. Conditional Approach to Updating

The conditional approach to fusing evidence 'conditions' or 'updates' the already available evidence with respect to what both FoDs can discern [9]. Once the conditioning operation is performed, each source invokes a strategy to incorporate its originally cast evidence that does not belong to $\Theta_1 \cap \Theta_2$. This approach enables a source to update its own knowledge base, and exchange information with other sources for the express purpose of refining its own knowledge, without having to continually 'expand' its FoD.

1) CUE: Identical FoDs Case: The *conditional update equation (CUE)* for the identical FoDs case in [10] embraces this conditional approach. To explain, consider the two BoEs $\mathcal{E}_i[k]$ with $\Theta\equiv\Theta_1=\Theta_2$. The CUE in [10] then yields the update $\mathcal{E}_1[k+1]\equiv\mathcal{E}_1[k]\triangleleft\mathcal{E}_2[k]$, $\forall k\geq 0$, of $\mathcal{E}_1[k]$ as

$$Bl_{\Theta_1}(B)[k+1] = \alpha[k]\, Bl_{\Theta_1}(B)[k] + \sum_{A\subseteq\Theta_2} \beta(A)[k]\, Bl_{\Theta_2}(B|A)[k], \quad (1)$$

where $Bl_{\Theta_2}(A) > 0$ and the parameters $\{\alpha[\bullet], \beta(A)[\bullet]\}$ are non-negative, $\beta(A)[\bullet] = 0$, $\forall A \notin \mathfrak{F}_{\Theta_2}[\bullet]$, and

$$\alpha[k] + \sum_{A\subseteq\Theta_2} \beta(A)[k] = 1, \forall k \geq 0. \quad (2)$$

Henceforth, we will not explicitly identify the index k unless it is essential. The conditional operation in (1) is implemented using the Fagin-Halpern (FH) DS theoretic conditionals.

Definition 3: [11] For $\mathcal{E}=\{\Theta, \mathfrak{F}_\Theta, m_\Theta(\bullet)\}$, $A, B \subseteq \Theta$ with $Bl_\Theta(A) > 0$, the *conditional belief* of B given A is

$$Bl_\Theta(B|A) = Bl_\Theta(A\cap B)/[Bl_\Theta(A\cap B) + Pl_\Theta(A\setminus B)].$$

■

Remarks:

1. Because $Bl_\Theta(B|A) = Bl_\Theta(A\cap B|A)$, while evaluating the evidence we have in support of B when our view is restricted to only A, the FH conditionals consider only those propositions that both A and B have in common.

2. The FH conditionals give a more appropriate probabilistic interpretation and a natural transition to Bayesian notions [9], [11]. Indeed, it is the FH conditional belief and plausibility that correspond precisely to the inner and outer measures of a non-measurable event [12]. See [9] for a detailed interpretation.

III. CUE: Non-Identical FoDs Case

Fig. 1 depicts the non-identical FoDs case where $\Theta_1 \neq \Theta_2$ and $\Theta_1 \cap \Theta_2 \neq \emptyset$. Extending the initial work in [9], [13] has proposed a strategy that enables one to update a BoE with the 'occurrence' of a proposition A residing in another BoE having possibly a different FoD, i.e., the incoming evidence which may have a non-identical FoD has *only one* focal element. Taking inspiration from this work, we now extend the CUE in (1) to accommodate non-identical FoDs.

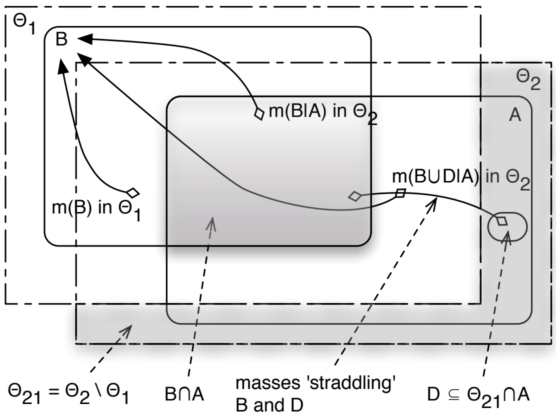

Fig. 1. Updating the BoE $\mathcal{E}_1 = \{\Theta_1, \mathfrak{F}_{\Theta_1}, m_{\Theta_1}(\bullet)\}$ with the evidence of BoE $\mathcal{E}_2 = \{\Theta_2, \mathfrak{F}_{\Theta_2}, m_{\Theta_2}(\bullet)\}$ when $\Theta_1 \neq \Theta_2$ and $\Theta_1 \cap \Theta_2 \neq \emptyset$. The terms that contribute towards the update of $m_{\Theta_1}(B)$ are also shown.

A. Belief CUE

Definition 4: For the BoEs $\mathcal{E}_i$, the CUE that updates $\mathcal{E}_1$ with the evidence in $\mathcal{E}_2$ is $\mathcal{E}_1[k+1] \equiv \mathcal{E}_1 \lhd \mathcal{E}_2$, $\forall k \geq 0$, where

$$\begin{aligned} Bl_{\Theta_1}(B)[k+1] &= \alpha\, Bl_{\Theta_1}(B) + \sum_{A \subseteq \Theta_2} \frac{\beta(A)}{2} \Big\{ Bl_{\Theta_2}(B|A) \\ &\quad + Bl_{\Theta_2}(D_B|A) - Bl_{\Theta_2}(\Theta_{21}|A) \Big\}, \end{aligned}$$

where $\Theta_{21} = \Theta_2 \setminus \Theta_1$, $D_B = B \cup \Theta_{21}$, and $Bl_{\Theta_2}(A) > 0$. The CUE parameters $\{\alpha, \beta(A)\}$ are non-negative, $\beta(A) = 0, \forall A \notin \mathfrak{F}_{\Theta_2}$, and

$$\alpha + \sum_{A \subseteq \Theta_2} \frac{\beta(A)}{2} \Big\{ Bl_{\Theta_2}(\Theta_1|A) + Pl_{\Theta_2}(\Theta_1|A) \Big\} = 1. \quad \blacksquare$$

Remarks:

1. Note that, $Bl_{\Theta_1}(\emptyset) = 0$ and $Bl_{\Theta_1}(\Theta_1) = 1$.

2. The focal elements $A \in \mathfrak{F}_{\Theta_2}$ that do not intersect with Θ_1 bears no influence on the CUE (even if $\beta(A) > 0$ for such propositions). Therefore, without loss of generality, we can assume that $\beta(A) = 0, \forall A \in \mathfrak{F}_{\Theta_2}$ s.t. $A \cap \Theta_1 = \emptyset$.

3. $Bl_{\Theta_1}(B)$ accounts for the evidence that $\mathcal{E}_1$ already has for $B \subseteq \Theta_1$; $Bl_{\Theta_2}(B|A)$ accounts for the conditional evidence that $\mathcal{E}_2$ has for propositions in $\Theta_1 \cap \Theta_2$; the terms $Bl_{\Theta_2}(D_B|A) - Bl_{\Theta_2}(\Theta_{21}|A)$ account for conditional evidence that $\mathcal{E}_2$ has for propositions that 'straddle' B and Θ_{21}.

4. $\Theta_2 \subseteq \Theta_1$ Case: When Θ_2 is contained in Θ_1, the belief CUE in Definition 4 reduces to

$$Bl_{\Theta_1}(B)[k+1] = \alpha\, Bl_{\Theta_1}(B) + \sum_{A \subseteq \Theta_2} \beta(A)\, Bl_{\Theta_2}(B|A), \quad (3)$$

where $\alpha + \sum_{A \subseteq \Theta_2} \beta(A) = 1$ with $\beta(A) = 0, \forall A \notin \mathfrak{F}_{\Theta_2}$. The strategy in [10] where $\Theta_1 = \Theta_2$ is a further special case.

5. Henceforth, unless otherwise mentioned, we do not consider the 'trivial' parameter values of $\alpha = 0$ and $\alpha = 1$.

B. Mass CUE

An equivalent statement of the belief CUE is

Lemma 1: The mass CUE corresponding to the belief CUE in Definition 4 is

$$\begin{aligned} m_{\Theta_1}(B)[k+1] &= \alpha\, m_{\Theta_1}(B) + \sum_{B \subseteq A \subseteq \Theta_2} \frac{\beta(A)}{2} \Big\{ m_{\Theta_2}(B|A) \\ &\quad + \sum_{D \subseteq \Theta_{21} \cap A} m_{\Theta_2}(B \cup D|A) \Big\}. \quad \square \end{aligned}$$

Proof: We know that

$$Bl_{\Theta_1}(B) = \sum_{C \subseteq B} m_{\Theta_1}(C), \;\; Bl_{\Theta_2}(B|A) = \sum_{C \subseteq B} m_{\Theta_2}(C|A),$$

and

$$\begin{aligned} Bl_{\Theta_2}(D_B|A) &= \sum_{D \subseteq \Theta_{21} \cap A} m_{\Theta_2}(D|A) \\ &\quad + \sum_{\emptyset \neq C \subseteq B} \sum_{D \subseteq \Theta_{21} \cap A} m_{\Theta_2}(C \cup D|A), \end{aligned}$$

where we used the fact that $m_{\Theta_2}(B|A) \neq 0$ only when $B \subseteq A$ [9]. Substitute these into the belief CUE and use the fact that it is true for all $B \subseteq \Theta_1$ to establish the claim. $\blacksquare$

Fig. 1 shows the masses that contribute towards updating $m_{\Theta_1}(B)$.

C. Focal Elements

The CUE in Definition 4 and Lemma 1 can generate 'new' focal elements that are not necessarily in $\mathfrak{F}_{\Theta_1} \cup \mathfrak{F}_{\Theta_2}$.

Lemma 2: For $B \in \mathfrak{F}_{\Theta_1}[k+1]$, either (i) $B \in \mathfrak{F}_{\Theta_1} \cup \mathfrak{F}_{\Theta_2}$; or (ii) $\exists\, A \subseteq \Theta_2$ s.t. $\emptyset \neq B \subseteq A$, $\beta(A) > 0$, and $Bl_{\Theta_2}(B \cup D) > 0$ for some $D \subseteq \Theta_{21} \cap A$. $\square$

Proof:

(i) It is obvious that $B \in \mathfrak{F}_{\Theta_1}$ implies $B \in \mathfrak{F}_{\Theta_1}[k+1]$. If $B \in \mathfrak{F}_{\Theta_2}$, $m_{\Theta_2}(B|B) > 0$ [9] and hence $B \in \mathfrak{F}_{\Theta_1}[k+1]$.

(ii) Assume $B \notin \mathfrak{F}_{\Theta_1} \cup \mathfrak{F}_{\Theta_2}$. Then $B \in \mathfrak{F}_{\Theta_1}[k+1]$ requires that $\exists\, A \subseteq \Theta_2$ s.t. $B \subseteq A$ with $\beta(A) > 0$; otherwise, all the terms in the right-hand side of the mass CUE vanish. With such an A, suppose $Bl_{\Theta_2}(B \cup D) = 0, \forall D \subseteq \Theta_{21} \cap A$. Then $Bl_{\Theta_2}(B \cup D|A) = 0$ implying that $m_{\Theta_2}(G|A) = 0, \forall G \subseteq B \cup D$. This would again render all the terms in the right-hand side of the mass CUE vanish. So, we must have $Bl_{\Theta_2}(B \cup D) > 0$ for some $D \subseteq \Theta_{21} \cap A$. $\blacksquare$

D. Selection of Parameters

1) Selection of α: The work in [9], [13] elaborates upon how α can capture the flexibility of available evidence towards changes (e.g., when the 'inertia' of the available evidence makes the source reluctant to change). Instead of repeating these strategies here, we refer the reader to [9], [13].

2) Selection of $\beta(A)$: Immediately, we see two choices.

Definition 5:

(i) *Receptive updating (rCUE):* $\beta(A) = K_{\Theta_2} m_{\Theta_2}(A)$, $\forall A \in \mathfrak{F}_{\Theta_2}$, where $K_{\Theta_2} \neq 0$ is a constant.

(ii) *Cautious updating (cCUE):* $\beta(A) = K_{\Theta_1} m_{\Theta_1}(A \cap \Theta_1)$, $\forall A \in \mathfrak{F}_{\Theta_2}$, where $K_{\Theta_1} \neq 0$ is a constant. $\blacksquare$

Remarks:

1. rCUE 'weighs' the incoming evidence according to $\mathcal{E}_2$'s support for each focal element. However, the focal elements contained in Θ_{21} would not contribute towards the update.

2. cCUE 'weighs' the incoming evidence according to $\mathcal{E}_1$'s support for each focal element. So the focal elements of the updated BoE are restricted to a subset of $\mathfrak{F}_{\Theta_1}$.

One can also take a more 'ad hoc' approach and choose $\beta(A)$ to emphasize, to different degrees, $\mathcal{E}_2$'s evidence towards selected propositions. For example, the choice of $\beta(\Theta_2) = 0$ enables $\mathcal{E}_1$ to completely disregard $\mathcal{E}_2$'s 'evidence' towards complete ignorance.

IV. Some Properties of the CUE

We now present several interesting and intuitively appealing properties of the CUE. Their formal proofs are not too difficult; but they are omitted to contain the length of this paper.

A. Mass Update for Complete Ambiguity

Claim 3: The CUE updates the complete FoD Θ_1 as

$$m_{\Theta_1}(\Theta_1)[k+1] = \alpha\, m_{\Theta_1}(\Theta_1) + \sum_{\Theta_1 \subseteq A \subseteq \Theta_2} \frac{\beta(A)}{2} \Big\{ m_{\Theta_2}(\Theta_1|A) + \sum_{D \subseteq \Theta_{21} \cap A} m_{\Theta_2}(\Theta_1 \cup D|A) \Big\}. \quad \blacksquare$$

Remarks:

1. For $\Theta_1 = \Theta_2$, Claim 3 reduces to

$$m_{\Theta_1}(\Theta_1)[k+1] = \alpha\, m_{\Theta_1}(\Theta_1) + \beta(\Theta_2)\, m_{\Theta_2}(\Theta_2). \quad (4)$$

2. For $\Theta_1 \not\subseteq \Theta_2$, Claim 3 reduces to

$$m_{\Theta_1}(\Theta_1)[k+1] = \alpha\, m_{\Theta_1}(\Theta_1) < m_{\Theta_1}(\Theta_1). \quad (5)$$

B. Updating a Vacuous BoE with an Arbitrary BoE

Claim 4: Consider the update $\mathcal{E}_1[k+1] = \mathcal{E}_1 \lhd \mathcal{E}_2$, where $\mathcal{E}_1$ is vacuous. Then, $\forall A \in \mathfrak{F}_{\Theta_2}$ s.t. $\Theta_1 \cap A \neq \emptyset$ and $\beta(A) > 0$, $m_{\Theta_1}(\Theta_1 \cap A)[k+1] > 0$. $\blacksquare$

Remark: So, whenever $\exists\, A \in \mathfrak{F}_{\Theta_2}$ s.t. $\emptyset \neq \Theta_1 \cap A \subset \Theta_1$ and $\beta(A) > 0$, $m_{\Theta_1}(\Theta_1)[k+1] < 1$ so that $\mathcal{E}_1[k+1]$ is no longer vacuous, an intuitively very appealing property.

C. Updating an Arbitrary BoE with a Vacuous BoE

Claim 5: Consider the update $\mathcal{E}_1[k+1] = \mathcal{E}_1 \lhd \mathcal{E}_2$, where $\mathcal{E}_2$ is vacuous. Then the mass CUE is

$$m_{\Theta_1}(B)[k+1] = \alpha\, m_{\Theta_1}(B) + \frac{\beta(\Theta_2)}{2} \Big\{ m_{\Theta_2}(B|\Theta_2) + m_{\Theta_2}(B \cup \Theta_{21}|\Theta_2) \Big\},$$

where $\alpha + \dfrac{\beta(\Theta_2)}{2} \left\{ Bl_{\Theta_1}(\Theta_1|\Theta_2) + Pl_{\Theta_1}(\Theta_1|\Theta_2) \right\} = 1.$ $\blacksquare$

Remarks:

1. For $\Theta_1 \subseteq \Theta_2$, Claim 5 reduces to

$$m_{\Theta_1}(B)[k+1] = \begin{cases} \alpha\, m_{\Theta_1}(B) < m_{\Theta_1}(B), \\ \qquad \text{for } B \subset \Theta_1; \\ \alpha\, m_{\Theta_1}(\Theta_1) + (1-\alpha) > m_{\Theta_1}(\Theta_1), \\ \qquad \text{for } B = \Theta_1. \end{cases}$$

2. For $\Theta_1 \not\subseteq \Theta_2$, Claim 5 reduces to

$$m_{\Theta_1}(B)[k+1] = \begin{cases} \alpha\, m_{\Theta_1}(\Theta_1 \cap \Theta_2) + (1-\alpha) \\ \qquad > m_{\Theta_1}(\Theta_1 \cap \Theta_2), \\ \qquad \text{for } B = \Theta_1 \cap \Theta_2; \\ \alpha\, m_{\Theta_1}(B) < m_{\Theta_1}(B), \\ \qquad \text{for } B \subseteq \Theta_1 \setminus (\Theta_1 \cap \Theta_2). \end{cases} \quad (6)$$

3. When $\mathcal{E}_1$ and $\mathcal{E}_2$ are *both* vacuous, Remarks 1-2 yield
—for $\Theta_1 \subseteq \Theta_2$: $\mathcal{E}_1[k+1]$ remains vacuous;
—for $\Theta_1 \not\subseteq \Theta_2$: the CUE generates

$$m_{\Theta_1}(B)[k+1] = \begin{cases} 1-\alpha, & \text{for } B = \Theta_1 \cap \Theta_2; \\ \alpha, & \text{for } B = \Theta_1. \end{cases} \quad (7)$$

While the $\Theta_1 \subseteq \Theta_2$ case above certainly does make intuitive sense, the $\Theta_1 \not\subseteq \Theta_2$ case calls for a careful examination. To elaborate, suppose $\mathcal{E}_1$ is vacuous with $\Theta_1 = \{a, b\}$. With $\Theta_2 = \{b, c, d, \ldots\}$, $|\Theta_2| >> |\Theta_1|$, suppose $\mathcal{E}_2$ is vacuous, $\mathcal{E}_2'$ has $m_2'(bcde) = 1.0$, and $\mathcal{E}_2''$ has $m_2''(bc) = 1.0$. With cCUE, $\alpha = 1$ and all the updates $\mathcal{E}_1[k+1]$, $\mathcal{E}_1'[k+1]$, and $\mathcal{E}_1''[k+1]$ will remain vacuous. This is what one would get with ballooning extension methods too [8]. But with rCUE, $\alpha < 1$ and all the updates will get $\{b\}$ as a focal element. While this may be questionable for $\mathcal{E}_1[k+1]$ and $\mathcal{E}_1'[k+1]$, it may in fact be acceptable, even desirable, for $\mathcal{E}_1''[k+1]$ to have $\{b\}$ as a focal element. So, what one deems reasonable may depend on how the parameters are selected and, in turn, the application. The strength of the CUE lies in its flexibility to provide a customized updating scheme via the choice of $\beta(\centerdot)$. For instance, with $\beta(\Theta_2) = 0$, one can disregard the incoming BoE's 'evidence' towards complete ignorance; or, one can pick $\beta(A) = K_2 m_2(A)$, $\forall A \in \mathfrak{F}_2$, s.t. $|A \setminus (\Theta_1 \cap \Theta_2)| \leq N$, and $\beta(A) = 0$, otherwise. With $N = 1$, then $\mathcal{E}_1''[k+1]$ will have $\{b\}$ as a focal element, but $\mathcal{E}_1[k+1]$ or $\mathcal{E}_1'[k+1]$ will not.

D. CUE for the Probabilistic Case

Claim 6: When all the focal elements of both $\mathcal{E}_1$ and $\mathcal{E}_2$ are singletons, the CUE reduces to

$$P_{\Theta_1}(B)[k+1] = \begin{cases} \alpha\, P_{\Theta_1}(B), & \text{for } B \in \Theta_1 \setminus \Theta_2; \\ \alpha\, P_{\Theta_1}(B) + \beta(B), & \text{for } B \in \Theta_1 \cap \Theta_2, \end{cases}$$

where $\alpha + \sum_{B \in \Theta_1 \cap \Theta_2} \beta(B) = 1.$ $\blacksquare$

Remarks:

1. For $B \in \Theta_1 \setminus \Theta_2$, $P_{\Theta_1}(B)[k+1] < P_{\Theta_1}(B)$.

2. For $B \in \Theta_1 \cap \Theta_2$ (i.e., events that are in common with both Θ_1 and Θ_2), $P_{\Theta_1}(B)[k+1] > P_{\Theta_1}(B)$ iff $\beta(B) > (1-\alpha)\, P_{\Theta_1}(B)$, i.e., the probability of a common event B

can increase if the corresponding $\beta(B)$ is not too small. Let us study the rCUE and cCUE parameter selection strategies:

—rCUE: Here $\beta(B) = (1-\alpha)\, P_{\Theta_2}(B|\Theta_1 \cap \Theta_2)$. This yields $P_{\Theta_1}(B)[k+1] > P_{\Theta_1}(B)$ iff $P_{\Theta_2}(B|\Theta_1 \cap \Theta_2) > P_{\Theta_1}(B)$.

—cCUE: Here $\beta(B) = (1-\alpha)\, P_{\Theta_1}(B|\Theta_1 \cap \Theta_2)$. This yields $P_{\Theta_1}(B)[k+1] > P_{\Theta_1}(B)$ iff $P_{\Theta_1}(B|\Theta_1 \cap \Theta_2) > P_{\Theta_1}(B)$, which is of course trivially true, except when $P_{\Theta_1}(B) = 0$ which yields $P_{\Theta_1}(B)[k+1] = 0$.

E. Updating Contradictory BoEs

With $\Theta_1 = \Theta_2 = \{a, b\}$, consider the two sources

$$\begin{aligned} \mathcal{E}_1 &: \{m_{\Theta_1}(a), m_{\Theta_1}(\Theta_1)\} = \{\mu, 1-\mu\}; \\ \mathcal{E}_2 &: \{m_{\Theta_2}(b), m_{\Theta_1}(\Theta_1)\} = \{\nu, 1-\nu\},\ \nu < \mu < 1. \end{aligned} \tag{8}$$

Then the rCUE-update $\mathcal{E}_1 \lhd \mathcal{E}_2$ yields the 'odds ratio'

$$\begin{aligned} \frac{m_{\Theta_1}(a)[k+1]}{m_{\Theta_1}(b)[k+1]} &= \frac{\alpha\, \mu}{\beta(b) + \beta(a,b)\, \nu} = \frac{\alpha}{1-\alpha}\, \frac{\mu}{\nu\,(2-\nu)} \\ &\to \alpha/(1-\alpha),\ \text{as } \nu \to 1 \text{ with } \nu < \mu < 1. \end{aligned} \tag{9}$$

Here $\alpha + \beta(b) + \beta(a,b) = 1$. In contrast, the DCR-generated odds ratio tends to ∞ (irrespective of how close to one ν is). Clearly, the CUE behaves more reasonably in this scenario.

V. An Example

The selection of the most appropriate update strategy and its parameters is a non-trivial application dependent task. We illustrate the use of variants of CUE and give insights into parameter selection via the following hypothetical scenario.

A. Set-Up

A remote military storage facility, protected by an automated surveillance system, uses a suite of hard sensors $\mathcal{S}_H$ and soft evidence $\mathcal{S}_S$ in the form of ground intelligence for identification of 'objects' crossing its perimeter. The objects crossing the perimeter are classified into one of the four classes, $S{=}Soldier$, $F{=}FighterJet$, $T{=}Tank$, and $O{=}Other$. Each class, except O which accounts for an object that cannot be further sub-classified (e.g., an animal), may further be sub-classified as $F{=}Friendly$ or $E{=}Enemy$. So the exhaustive set of objects of interest is $\Theta_{Obj}{=}\{\underbrace{S_F, S_E}_{\equiv S}, \underbrace{F_F, F_E}_{\equiv F}, \underbrace{T_F, T_E}_{\equiv T}, O\}$.

1) Hard Evidence: Each sensor belonging to $\mathcal{S}_H$ can identify ground objects, but cannot differentiate between friendly and enemy forces. So, we capture the BoE associated with the sensor suite $\mathcal{S}_H$ via $\mathcal{E}_H{=}\{\Theta_H, \mathcal{F}_{\Theta_H}, m_{\Theta_H}(\bullet)\}$, where $\Theta_H{=}\{S, T, O\}$.

2) Soft Evidence: The prevailing threat level (TL) in the proximity of the security zone, military domain expert opinion, etc., constitute ground intelligence providing soft evidence on how one may view, and further refine, the evidence that hard sensors provide. We capture the BoE associated with soft evidence via $\mathcal{E}_S{=}\{\Theta_S, \mathcal{F}_{\Theta_S}, m_{\Theta_S}(\bullet)\}$, where $\Theta_S{=}\{S, T, F\}$.

We 'partition' the time axis into regions so that the disposition of the evidence being received within each region is unchanged. For our simulations, we use 4 such regions, (a), (b), (c), and (d). For convenience of identifying the appropriate BoE, we will utilize an index k to denote the the corresponding region (i.e., (a), (b), (c), or (d)); $k{+}1$ and $k{-}1$ denote the next and previous regions, respectively. For example, $\mathcal{E}_H[k = b]$ is the hard evidence generated within region (b); $\mathcal{E}_H[k-1]$ then identifies the hard evidence generated in region (a). The 'initial' region where no evidence has yet been received is modeled via the DS theoretic vacuous mass assignment. The evidence being generated by $\mathcal{S}_H$ and $\mathcal{S}_S$ within each distinct region is modeled via DS theoretic mass assignments and shown in Table I.

B. Fusion

In our simulations, we look at three schemes that a fusion center may employ to update its knowledge base. Let us denote the BoE generated at this fusion center via $\mathcal{E}_*[k]{=}\{\Theta_*, \mathfrak{F}_{\Theta_*}[k], m_{\Theta_*}(\bullet)[k]\}$, where Θ_* is the FoD being retained at the fusion center. We will assume that $\Theta_* = \Theta_H$.

1) Temporal Updating of Hard Evidence: Here, $\mathcal{E}_*[k+1]{=}\mathcal{E}_*[k] \lhd \mathcal{E}_H[k+1]$. Only the hard evidence is used; no soft evidence is incorporated. Since the fusion center has no evidence to begin with, we used rCUE with a low α (=0.3 in simulations). Fig 2 shows the fusion results. See how the support towards a Tank T is increased with temporal fusion.

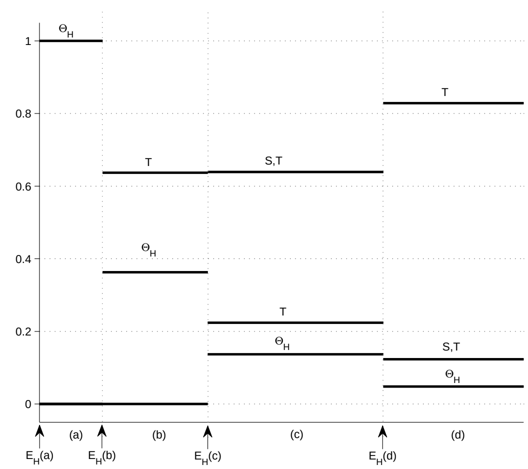

Fig. 2. Fusion results corresponding to temporal updating of hard evidence. rCUE is used with $\alpha = 0.3$ allowing a higher flexibility to change.

2) Modal Updating of Hard Evidence (with Soft Evidence): Here, $\mathcal{E}_*[k]{=}\mathcal{E}_H[k] \lhd \mathcal{E}_S[k]$. The hard evidence in a given region is simply updated by the soft evidence available within the same region; no temporal updating is incorporated. With no additional information regarding the credibility of the soft source, we used cCUE in this simulation. Of course, when a hard sensor fails or in the initial stages of evidence collection (e.g., in region (a)), rCUE may be more suitable. See Fig 3 for fusion results. Notice how the fusion center's supports get more refined with the incorporation of soft evidence. For instance, see region (c): although the hard evidence assigns more mass to T and (S, T) (simply because it cannot differentiate F and E sub-classes), fused results clearly indicate a redistribution of masses into sub-classes.

TABLE I

DS THEORETIC MASS ASSIGNMENT MODELS OF THE EVIDENCE GENERATED BY $\mathcal{S}_H$ AND $\mathcal{S}_S$

BoE	Region (a)	Region (b)	Region (c)	Region (d)
$\mathcal{E}_H(\centerdot)$ (Hard)	$\{\Theta_H\} = \{1.0\}$	$\{T, \Theta_H\} = \{0.7, 0.3\}$	$\{T, \Theta_H \setminus O, \Theta_H\} = \{0.4, 0.4, 0.2\}$	$\{T, \Theta_H \setminus O, \Theta_H\} = \{0.7, 0.2, 0.1\}$
$\mathcal{E}_S(\centerdot)$ (Soft)	$\{F_E, \Theta_S\} = \{0.8, 0.2\}$	$\{F_E, (S, F_E, T_E), \Theta_S\} = \{0.8, 0.1, 0.1\}$	$\{(S, F_E, T_E), (S_F, F_E, T_E), \Theta_S\} = \{0.4, 0.5, 0.1\}$	$\{(S_F, T_E), \Theta_S\} = \{0.9, 0.1\}$

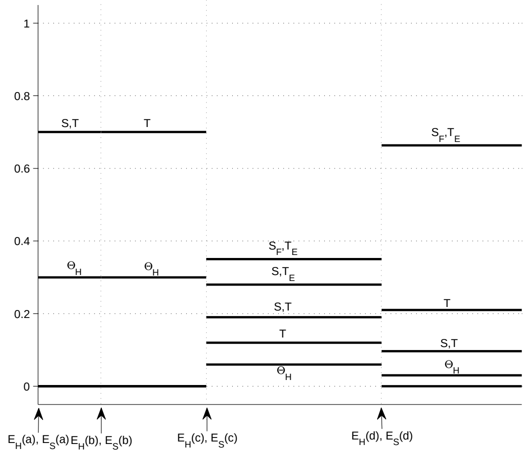

Fig. 3. Fusion results corresponding to modal updating of hard evidence. rCUE is used for region (a); cCUE is used for all other regions with α=0.3.

3) Modal Updating Followed by Temporal Updating of Hard Evidence: Here, $\mathcal{E}_*[k+1]=\mathcal{E}_*[k] \lhd (\mathcal{E}_H[k+1] \lhd \mathcal{E}_S[k+1])$, i.e., this scheme combines the two schemes previously mentioned. Fusion center updates itself by first refining the hard evidence using soft evidence. In such a scenario, one may select α to reflect the integrity of the existing knowledge. Fig 4 shows the fusion results.

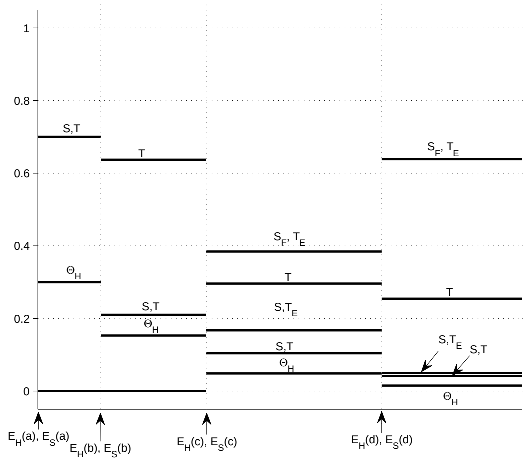

Fig. 4. Fusion results corresponding to modal updating followed by temporal updating of hard evidence. Modal update is done as in Section V-B.2; temporal update is done via rCUE with α=0.3.

VI. CONCLUDING REMARKS

The CUE possesses several intuitively appealing features which seem to indicate its suitability for scenarios that call for soft and hard evidence fusion. In particular, it can accommodate sources possessing non-identical scopes and it performs reasonably well when confronted with contradictory evidence.

Among various issues that warrant further investigation, of particular importance is the computational complexity that hampers the use of DS theoretic methods when working with a high number of sources and/or source FoDs having high cardinality. The conditional approach certainly helps because it does not require one to work with an 'expanded' FoD. However, much more has to be done to lighten the computational burden. The use of special mass assignments (e.g., Dirichlet structure [10]) is one strategy that we are currently studying.

Given that evidence updating is in general not 'commutative', another issue that requires attention is how one may 'sequence' the evidence sources.

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