Approximation in DSm theory for fusing ESM reports

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Abstract— **Electronic Support Measures consist of passive receivers which can identify emitters which, in turn, can be related to platforms that belong to 3 classes: Friend, Neutral, or Hostile. Decision makers prefer results presented in STANAG 1241 allegiance form, which adds 2 new classes: Assumed Friend, and Suspect. Dezert-Smarandache (DSm) theory is particularly suited to this problem, since it allows for intersections between the original 3 classes. However, as we know, the DSm hybrid combination rule is highly complex to execute and requires high amounts of resources. We have applied and studied a Matlab implementation of Tessem's klx approximation technique in the DSm theory for the fusion of ESM reports. Results are presented showing that we can improve on the time of execution while maintaining the same rate of good decisions.**

Keywords: Dezert-Smarandache Theory, ESM, klx, approximation.

I. INTRODUCTION

In terms of classification, the Dezert-Smarandache theory (DSmT) can become quite useful, especially for the direct resolution of classification for cases of hierarchical classes structures. For instance, we have the case of the allegiance classification structure suggested by STANAG 1241 where a structure of five classes (3 main classes and 2 derived classes) is required. The DSmT is able to output to any of those classes without modifications to its fusion process.

However, this example is still a simple one and both DSmT theories, with or without approximation, can solve it quite easily, which wouldn't be the case for classification problems of higher dimension. By dimension we mean the cardinal of the frame of discernment. In fact, the DSmT, as it is, can become highly complex and computationally prohibitive as soon as we reach a dimension of 6. That is a classification of a problem having six main classes and up to, in the worst case scenario, a total of 7,828,353 possible derived classes.

Various avenues of research have been tried to avoid or address this complexity problem [10, 13, 18]. However, even just counting the number of possible classes is still an active problem in mathematics known as the Dedekind problem, or the problem of counting antichains [9, 18].

In this paper, we considered using an approximation technique to restrain the staggering amount of data that the DSmT can generate in its fusion process. More specifically we have chosen Tessem's klx approximation technique [4] and used it into the DSmT with the DSm hybrid combination rule (DSmH). We have also experimented with the fusion process while using the approximation technique and compared it to the case without an approximation technique to analyze how it affects the quality of the decision process. More specifically, we will compare the good decision rate in the two cases, with and without the use of approximation.

A. Realistic Study Case

Electronic Support Measures (ESM) consist of passive receivers which can identify emitters coming from a small bearing angle, which, in turn, can be related to platforms that belong to 3 classes: either Friend (F), Neutral (N), or Hostile (H). Decision makers prefer results presented in STANAG 1241 allegiance form, which adds 2 classes: Assumed Friend (AF), and Suspect (S).

The DSm theory is particularly suited to this problem, since it allows for intersections between the original three classes of allegiance. In this way an intersection of Friend and Neutral can lead to an Assumed Friend, and an intersection of Hostile and Neutral can lead to a Suspect. This structure of allegiances will be referred to as STANAG allegiance [11].

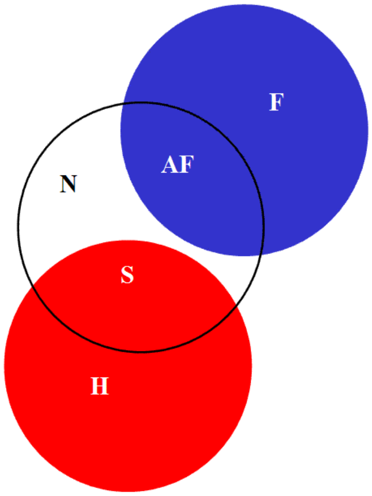

Figure 1. Venn diagram for the STANAG allegiances.

Figure 1 displays a visual representation of a possible interpretation of STANAG allegiance in DSmT. We can see that even though the input consists only of 3 classes, we are able to give an output into 5 classes. For example, here we have the class ‘Suspect’, which could be the result obtained after fusing ‘Hostile’ with ‘Neutral’. We also have the class ‘Assumed Friend’, which could be the result obtained after fusing ‘Friend’ with ‘Neutral’. Note that this case example has the intersection $F \cap H = \emptyset$, the null set, which is a constraint in DSm, leading to the use of its hybrid rule. This case example would be relevant for peace-keeping missions where Hostile and Friendly forces aren’t likely to be close one to another. We will be working on that case, with $F \cap H = \emptyset$.

II. Dezert-Smarandache Theory

The DSm theory uses the language of masses assigned to each declaration from a sensor (in our case, the ESM sensor). In DSm theory, all unions and intersections are allowed for a declaration. For our case of cardinality 3, $\Theta = \{\theta_1, \theta_2, \theta_3\}$, with $|\Theta| = 3$, D^Θ is still of manageable size, namely has a cardinality of 19 [10]. In DSm theory, a constraint like the one that was imposed by Figure 1, namely that $F \cap H \equiv \theta_1 \cap \theta_3 = \emptyset$ is treated by the DSm hybrid combination rule (DSmH) below:

$$m(A) = \phi(A) \left[S_1(A) + S_2(A) + S_3(A) \right] \tag{1}$$

The reader is referred to a series of books [10, 13, 17] on DSm theory for lengthy descriptions of the meaning of this formula (note that the function ϕ is not to be confused with the empty set). A three-step approach was proposed in [12], which is used here. The incoming sensor reports are either: Friend ($F=\theta_1$), Neutral ($N=\theta_2$) or Hostile ($H=\theta_3$), Figure 1 has the interpretation of the five classes:

$$\text{Friend} = \{\theta_1 - \theta_1 \cap \theta_2\} \tag{2}$$

$$\text{Hostile} = \{\theta_3 - \theta_3 \cap \theta_2\} \tag{3}$$

$$\text{Assumed Friend} = \{\theta_1 \cap \theta_2\} \tag{4}$$

$$\text{Suspect} = \{\theta_2 \cap \theta_3\} \tag{5}$$

$$\text{Neutral} = \{\theta_2 - \theta_1 \cap \theta_2 - \theta_3 \cap \theta_2\} \tag{6}$$

As in [15], we call STANAG-probability the pignistic probability assigned to the five classes shown by equations (2) to (6). We use the general pignistic transform, as shown by [10], to obtain the probability values of the sets used in those equations.

III. Approximation technique

The klx approximation technique developed by Tessem [4] is designed to approximate Basic Probability Assignment (BPA) or mass function in Dempster-Shafer Theory (DST). Since DSm theory works directly with BPAs, applying the klx approximation technique to the DSmH is quite straightforward and can be done without any changes. Figure 2 shows the algorithm of the klx approximation technique as described by Bauer [5] and as we’ve applied in our study within the DSm framework.

Note that we have also considered transferring to DSmT Bauer's D1 approximation technique [5]. However, when compared to klx, D1 would have a greater amount of operations to execute. It seemed counterproductive considering that the number of operations, or complexity, is a factor we want to reduce.

```
Function klx-approximation(m, k, l, x)
% parameters list:
% m : focal elements of bpa to approximate
% k : minimal number of focal elements to keep
% l : maximal number of focal elements to keep
% x : maximal mass to be removed

Sort m according to masses
Tmass = 0; % Total mass in approximated bpa ($m_{klx}$)
f = 0;     % Number of focal elements of $m_{klx}$
while (m contains focal elements) and ( $f \le l$ )
        and ( ($f < k$) or (Tmass < $1 - x$) ) do
   add next focal element A of m to $m_{klx}$
   f = f + 1;
   Tmass = Tmass + m(A);
end while
Normalize $m_{klx}$
             % by dividing focal elements’ mass by Tmass
return
```

Figure 2. klx approximation technique.

IV. A TYPICAL SIMULATION SCENARIO

The pre-requisites that a typical scenario must address are: (1) to be able to adequately represent the known ground truth, (2) to contain sufficient countermeasures (or miss-associations) to be realistic and to test the robustness of the theories, (3) to only provide partial knowledge about the ESM sensor declaration, which therefore contains uncertainty, (4) to be able to show stability under countermeasures, yet (5) to be able to switch allegiance when the ground truth does so.

The following scenario parameters have therefore been chosen accordingly: (1) ground truth is FRIEND for the first 50 iterations of the scenario and HOSTILE for the last 50, (2) the number of correct associations is 80%, corresponding to countermeasures appearing 20% of the time, in a randomly selected sequence, (3) the ESM declaration has a mass (confidence value in Bayesian terms) of 0.8, with the rest of the mass being assigned to the ignorance (the full set of elements, namely Θ).

This scenario will be the one addressed in the next section, while a Monte-Carlo study is described in the subsequent section. Each Monte-Carlo run corresponds to a different realization using the above scenario parameters, but with a different random seed. The scenario chosen is depicted in figure 3.

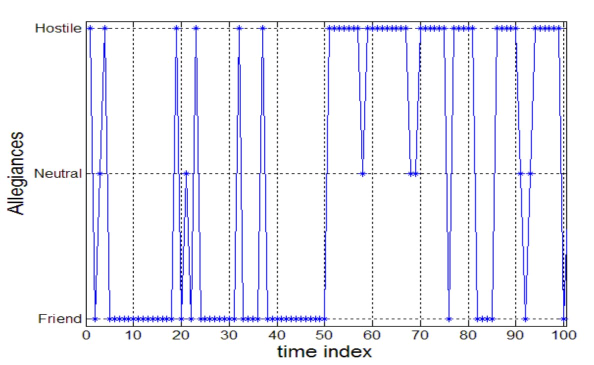

Figure 3: Chosen scenario.

Roughly 80% of the time the ESM declares the correct allegiance according to ground truth, and the remaining 20% is roughly equally split between the other two allegiances. Note that these percentages of occurrences are from a statistical point of view only, so that in the long run a large amount of randomly generated scenarios would amount to these ratios. There is an allegiance switch at the 50th time index, and the selected randomly selected seed in the above generated scenario generates a rather unusual sequence of 4 false Friend declarations starting at time index 82 (when actually Hostile is the ground truth).

A. Results for the simulated scenario

Before presenting the results, it should be noted that the original form of the DSmH tends to accumulates masses to intersections as is the case for any rule based on conjunction [14]. An ad hoc solution exists [3, 7, 8], and consists in renormalizing after each fusion step by giving a value to the complete ignorance which can never be below a certain factor (chosen here to be 0.04 as research in [14] shows that this value is appropriate for this case while being high enough to avoid the accumulation but still low enough not to interfere with the combination's performances). That solution was originally developed to the well-known problem of DST combination, which tends to be overly optimistic, which in turn prevents it to react quickly to changes of allegiances. For more on the behavior of the DSmH the reader is referred to [14, 15, 16], as we are focused on exploring the effect of approximation techniques on DSm in this paper.

Since the whole idea behind using DSm was to present the results to the decision maker in the STANAG allegiance format, the result of figure 4 would be used. For the DSmH [10], it was suggested to use the Generalized Pignistic Probability, which is based on the pignistic transformation [6], in order to make a decision on a singleton belonging to the input ESM-allegiance.

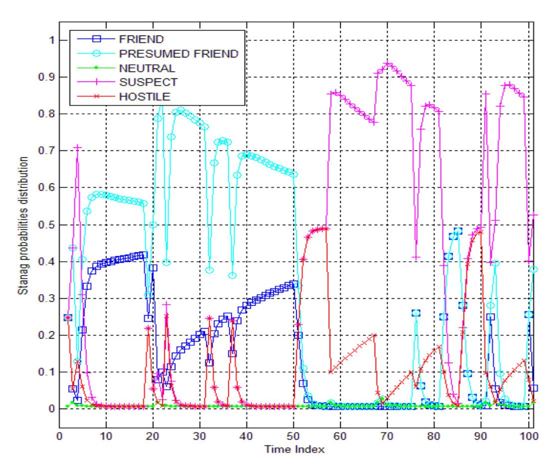

Figure 4: DSmH result for the chosen scenario.

The decision maker would clearly be informed that miss-associations have occurred, since Assumed Friend dominates for the first 50 time indices and Suspect for the latter 50. The Friend declarations starting at time index 82 cause confusion, as it should. The change in allegiance at time index 50 is detected quickly. What is even more important is that F and AF are clearly preferred for the first 50 time indexes and S and H for the last 50, as they should.

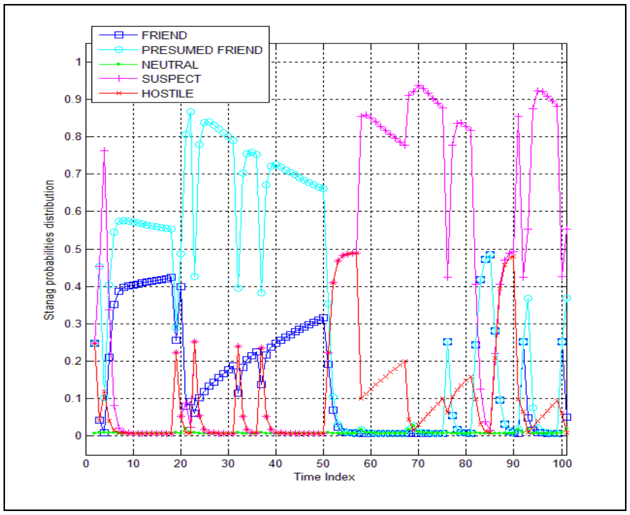

Figure 5: Approximated DSmH result for the same scenario with klx = (5, 6, 0.2).

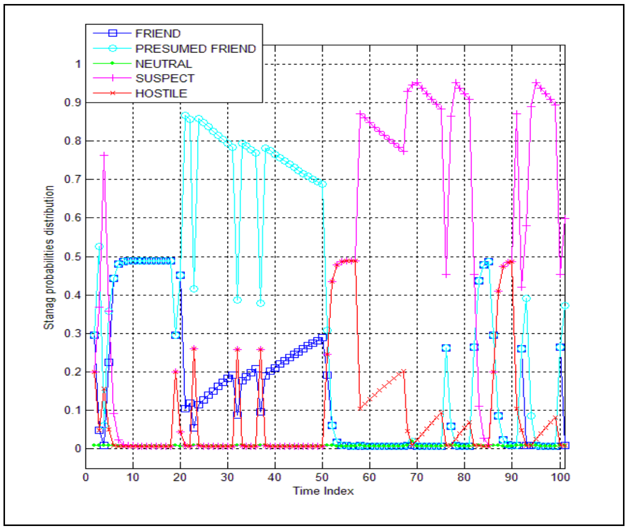

Figure 6: Approximated DSmH result for the same scenario with klx = (3, 6, 0.2).

We can gather from figures 4 and 5 that the DSmH and the approximated DSmH have very similar behaviors. In fact, one has to look at the figures very closely to perceive the differences. We can see that in the first half of the approximated version, the assumed friend allegiance is slightly favored to the friend allegiance. Near the end of the scenario the hostile allegiance is favored to the suspect allegiance. However, in both cases, even if the smallness of the change could possibly affect our decision, the STANAG-probability still seems to stay within the same type of allegiance in the sense that a friend and a target of assumed friend allegiance would both inspire a friendly response on our part. The same can be said for a target of suspect or hostile allegiance that would both inspire a hostile or defensive response on our part. In short, we can easily proceed with the approximation and still be able to make the same decision the same way.

B. *Effects of varying the klx parameters*

We've realized the scenario for various values of klx for k ∈ [3, 10], l ∈ [6, 12] and x ∈ [0.2, 0.4]. For the cases where we had k=8, no changes in l and x had impact, and compared to the DSmH, we've only noticed a very small variation at the start and end of the simulated scenario. For the cases where we had k=6, no changes in l and x had impact and compared to DSmH, there was only very little variation in value throughout the scenario. The same is true for the cases with k=5, with the figure 5 showing the results for that case. The amplitude of the variation between DSmH and the approximated version continues to increase as the k value diminishes.

We finally begin to notice small changes with x=0.2 as opposed to 0.3 or 0.4 when we reach k=4. However, the impact of having x at 0.2 is small and contained at the start of the scenario, where it gives more weight to the suspect class at the expense of the hostile class. For the cases with k=3, the impact of the change on x going to 0.2 was more significant and lasted throughout most of the scenario's duration. Also, while for cases of k ∈ [4, 8] the behavior of the curves were all very similar one to another, when we reach k=3, we observe a partial loss of smoothness, hence a more reactive behavior toward countermeasures and allegiance change. Figure 6 shows the case of the simulated scenario for an approximated DSmH with klx = (3, 6, 0.2). Note that in all our experimentations for our chosen scenario the l parameter never had any visible impact.

V. MONTE CARLO SIMULATIONS

Although a special case such as the one described in the previous section offers valuable insight, one might question if the conclusions from that one scenario pass the test of multiple Monte-Carlo scenarios. This question is answered in this section.

In order to expend the parameter space, we have realized the simulations of the current section to 80 and 90% for the ESM certainty, and with an ESM confidence at 80% and an ignorance threshold at 0.04 as before. The number of Monte-Carlo runs was set to 100. The randomly generated ESM stream of reports used for both the DSmH and the approximated DSmH are all the same so that we can freely compare the effects of the use of the approximation, and the impact of the variation of its parameters.

As for the choice of a the graphical display to highlight the results of our simulations, we went with the rate of good decisions, where a good decision is as we have mentioned earlier, when we conclude to be friendly toward a friendly behaving target, when the ground truth is of class friend. A friendly-behaving target is a target that is concluded to be a friend or an assumed friend. We also have a good decision when we conclude to be hostile toward a hostile behaving target, when the ground truth is of class hostile. A hostile-behaving target is a target that is concluded to be a hostile or a suspect. A decision is made by taking the set of maximum STANAG-probability.

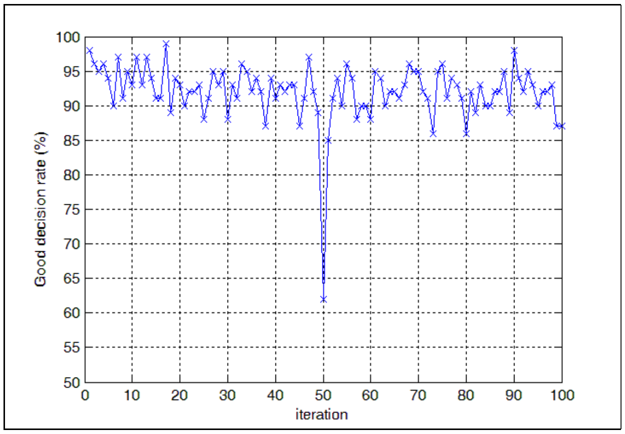

Figure 7. DSmH result after 100 Monte-Carlo runs.

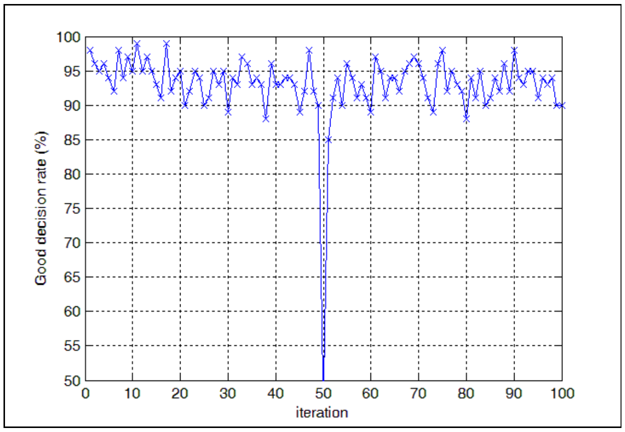

Figure 8. Approximated DSmH result with klx = (5, 8, 0.2) for the same Monte Carlo simulation.

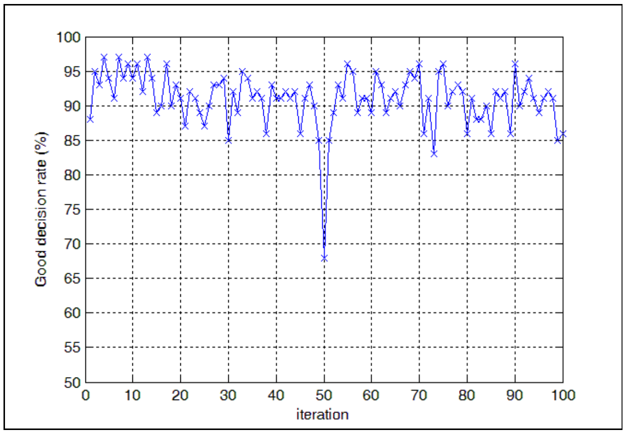

Figure 9. Approximated DSmH result with klx = (3, 8, 0.2) for the same Monte Carlo simulation.

A. *Effects of varying the klx parameters*

Simulations were done on a computer with an AMD Phenom II X4 955 processor having 8 GB of RAM. We should keep in mind that it is the relative time of execution which is important here. For figures 7 to 11, the simulations had a value of 80% for the ESM certainty and the value of the x parameter was maintained at 0.2 since changing it had no impact on good decision rate.

Figures 8 and 9 show us the effect of the approximation from the good decision rate point of view when compared with the DSmH case from figure 7. Like for the typical simulated scenario from previous section, l had no visible impact, and x had a limited impact only as the k parameter went below 4. As for the k parameter, it started having an impact when we reached 6, where the impact was on only three iterations. As the k parameter reaches 5, a very slight positive impact throughout the whole simulation can be seen. As for k=4 and k=3, we have a slight deterioration of the good decision rate but it is still very small and rather insignificant considering the gain in time execution as figure 11 shows us. For the cases with an ESM confidence at 90%, all the approximated results, have no significant impact on the good decision rate, except with klx = (3, 8, 0.2) where we had minimal impact.

We have the time of execution versus k and l parameters from the klx approximation technique on figures 10 and 11. Specifically, figure 10 has the curve of the time of execution of the combination and approximation process only. The x-y plane, valued at 325.97 seconds on figure 10 indicates the time from which the approximation process provides a higher gain in time than the time it consumes. It is the time of execution of the DSmH without approximation. We can see that the k parameter has to reach 5 before we start seeing an improvement. Before that value, the approximation takes more time to execute than it helps us gain. We can achieve a 30% improvement on time of execution when we reach k = 3. The parameter l has no impact on time. The absence of impact of the l parameter is suspected to be caused by the fact that this simulated scenario case uses simple support functions as inputs.

In figure 11, we have the curve of the time of execution for the whole simulation which, on top of the combination and approximation processes, includes the generalized pignistic transformation (GPT) which is used in the decision process. Above 95% of the extra time of execution, when compared to figure 10, is composed of the GPT. In figure 10, the x-y plane, representing the time of execution of the simulation without approximation, is valued at 1767.6 seconds. We can see that we can have a

50% reduction in time of execution when we reach k=3 and that l has no impact. As we compare figure 10 and 11, we see that the GPT is the step that benefits the most from the approximation process.

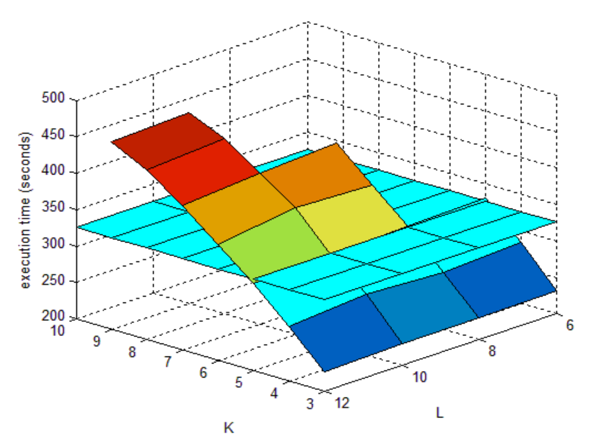

Figure 10. Execution time for the combination and approximation processes.

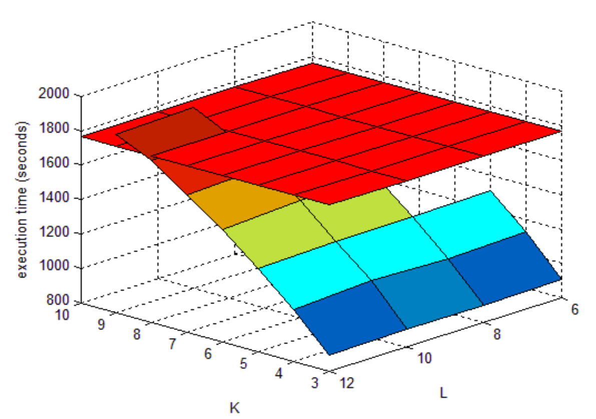

Figure 11. Execution time for the whole simulation.

VI. CONCLUSION

The previous section displays the behavior for different cases of klx approximation on the same simulated ESM data. It also shows the time of execution of each of those simulations. From our results we can conclude that we can successfully attain the same good decision rate with DSmH as with an approximated DSmH for the chosen scenario, while achieving lower times of execution including the time to approximate when we reach a certain level of approximation.

Future work considered includes the exploration of the use of Bauer's D1 approximation [5] in DSmT. Even if it adds to the number of operations and in the complexity of the system, it would be interesting to see if the gain acquired by approximating is sufficient to counter the increase in complexity.

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