Representation of parameter uncertainty with evidence theory in Probabilistic Risk Assessment

Tu Duong Le Duy[1], Dominique Vasseur[1]
Laurence Dieulle[2], Christophe Bérenguer [2], Mathieu Couplet[1]

[1]Risk Management Department / EDF R&D
1, avenue de Général de Gaulle, 92141 Clamart cedex, France
Email: tu-duong.le-duy@edf.fr, dominique.vasseur@edf.fr, mathieu.couplet@edf.fr
[2]Université de technologie de Troyes, Institut Charles Delaunay/LM2S
12, rue Marie Curie, BP2060, Troyes Cedex, France
Email: laurence.dieulle@utt.fr, christophe.berenguer@utt.fr

Abstract— In the current approach of nuclear power plant Probabilistic Risk Assessment, parameter uncertainty due to a lack of knowledge is generally represented by probability distribution (e.g. log-normal) whose mean or median is considered as point value estimated from operating experience feedback. In this paper, such an approach is shown to lead to potential erroneous and ambiguous results in decision making. To overcome this problem, the Dempster-Shafer Theory of Evidence is considered. In this paper, a so-called unified Dempster- Shafer representation which allows to deal with current issues is proposed to characterize the parameter uncertainty in an appropriate manner. The use of this representation is illustrated through a practical example of Probabilistic Risk Assessment.

Keywords: evidence theory**,** nuclear power plant Probabilistic Risk Assessment, parametric uncertainty.

I. INTRODUCTION

Probabilistic Risk Assessment (PRA) [1] is a methodology which provides a quantitative assessment of the risk of accidents at Nuclear Power Plants (NPP). It involves the development of models that delineate the response of systems and of operators to initiating events that could lead to core damage or a release of radioactivity to the environment. The evaluation of the frequency of such an accident relies on the assessment of failure probability of systems by means of event/fault tree analysis modelling component failures. However, in the context of NPP where available data are insufficient or imprecise, it is often difficult to estimate accurately the failure rates of individual components or the frequency of events. Therefore, this uncertainty needs to be taken into account in the decision making process.

In general, uncertainties can be categorized as either aleatory or epistemic. Aleatory uncertainty reflects our inability to predict random observable events, whereas epistemic uncertainty represents lack of knowledge with respect to the models or to the appropriate values to use for quantities that are assumed to be fixed but poorly known in the context of a particular analysis [2]. Many recent researches have shown that uncertainties in PRA are mainly epistemic [1]. Epistemic uncertainties can be roughly split into three categories as done in [1]. These are: parameter, model, and completeness uncertainties. In this article, we focus mainly on parameter uncertainty.

In the current PRA practice, parameter uncertainty representation relies on probability theory by using a single subjective probability distribution, such as a log-normal one. Nevertheless, the choice of this distribution is somehow arbitrary and mainly made because of conventional and historical reason. Moreover, numerous authors conclude that there are limitations in using probability theory to represent epistemic uncertainty [3][4]. A number of alternative representation frameworks, e.g. evidence theory, possibility theory (or fuzzy intervals) or interval analysis [4], have been proposed to represent epistemic uncertainty in a more appropriate manner. Evidence and possibility theories, in particular, may be the most attractive ones for risk assessment, because of their mathematical representation power.

In the section II of this paper, we will show that the traditional probabilistic approach for parameter uncertainty representation in reliability system can lead to erroneous results, thus wrong decision. A brief presentation of the Dempster-Shafer approach will be proposed in section III. The representation of parameter uncertainty within evidence theory on the basis of a mean or median associated to maximal and minimal values will be considered in section IV. The section V will propose a unified Dempster-Shafer representation to characterize the parameter uncertainty in PRA model. Finally, some results and conclusions on the application of this theory will be discussed in the final section through a practical PRA example.

II. CHARACTERIZATION OF PARAMETER UNCERTAINTY BY THE PROBABILITY THEORY

In the early stage, PRA models are built as probabilistic models to reflect the random nature of the constituent basic events such as initiating events and component failures. Randomness is one manifestation of a form of uncertainty which is often called *aleatory uncertainty*. A PRA is, therefore, a probabilistic model that characterizes the aleatory uncertainty associated with accidents at nuclear power plants [1]. Some probabilistic models like Poisson model, exponential model, and so on, are used for this purpose. The model parameters (ex. failure rate or initiating event frequency) are assumed to be constant over the time. The values of those (assumed) fixed but unknown parameters can be estimated using appropriate data. The two approaches widely used in PRA are the Bayesian and the frequentist ones. In the first method [1][5], the parameters are considered as random variables: they are associated with a prior distribution of probability which models the epistemic uncertainty. The posterior distribution is obtained by applying the Bayes formulae according to the available data. The application of the second approach, which is often chosen in [6], provides generally a point estimate obtained from Maximum Likelihood Estimation (MLE) and a confidence interval. In this method, when the data are sufficiently available, the point estimate of the estimator is approximately equal to the true value of the parameter. However, when available data are poor, the confidence interval is extremely large, being the expression of the insufficiency of the data. This type of uncertainty can be referred to lack of knowledge (or epistemic uncertainty). On the one hand, the 90% confidence interval obtained in PRA is expressed in terms of percentiles of a chi squared distribution. On the other hand, the current PRA practice [5][6] assumes widely a different distribution, that is to say the log-normal one, to take into account the parameter epistemic uncertainty. The distribution is then parameterised by the point estimate as the mean value and what is called the error factor. The use of the log-normal distribution is somehow arbitrary and widely assumed in PRA mainly because of conventional and historical reasons. This representation could be interpreted as a subjective probability assigned to each value in the confidence interval. The subjective probability is considered as the degree of belief that the parameter could take in this interval. However, as we can see later the use of this log-normal distribution can lead to an erroneous decision in risk analysis.

A. Limits of the log-normal distribution

In the following example, we consider a simple system of two identical motor operated valves (MOV) in parallel. The system unavailability point estimate Q is calculated in terms of each MOV failure probability p (per demand) by

$$Q = p^2 \quad (1)$$

This formula takes into account the state of knowledge correlation for identical components that are modelled by the same parameter in PRA model [7]. The uncertainty on each failure probability is modelled by a log-normal distribution with the error factor (EF) and the mean associated. As a result, the output variable Q follows a log-normal distribution as well (product of log-normal distributed variables). Now we are interested in assessing the probability that the output system unavailability exceeds a specified threshold θ, say $\theta = 1E-6$. Historically, it is typical to use assurance levels of 0.95 (95% percentile) as being characteristic of acceptability. It has been shown in [8] that if the failure probability of each component p given in data book has the point estimate value which is inferior or equal to $0.258\sqrt{\theta}$ = 2.58 E-4, the 95% percentile of the output distribution will always be inferior or equal to $\theta = 1E-6$ for all input error factors. This means that we have always at least 95% chance that the output failure probability of this system will not exceed the threshold. As a result, the decision maker doesn't need to care of uncertainties associated with input estimate values. However, what would happen if we assumed other probabilistic distributions. These conclusions could be no longer assured.

B. Impact of the meaning given to estimated value on decision making results

Up to now, in PRA for NNP there is still an ambiguous interpretation of the point estimate which is used to represent statistic quantity of log-normal distribution. In early WASH-1400 database, the best estimate (point estimate) is presented as a median value while it is recommended to interpret it as a mean value in recent NUREG-1150 PRA database because the mean value is higher than median value [1][9]. Apostolakis [7] has proposed to use the median rather than the mean value because the later is very sensitive to the tails of the distributions while the percentiles are not. This insensitivity makes the uses of percentiles preferable when we intend to measure the probability for a specific indicator to be greater than a given safety goal. While a robust and appropriate choice hasn't been established yet, the following example shows that, the choice of median or mean value for input uncertainty representation can lead to very different results for the decision making.

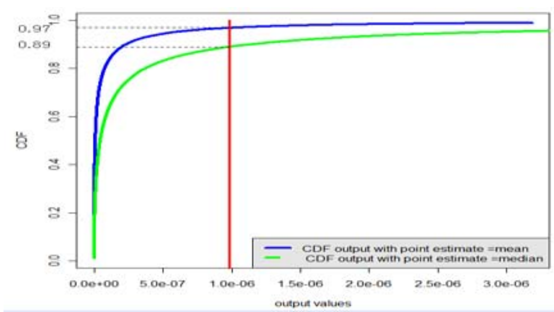

Figure 2. Output cumulative distribution functions (CDF) in two cases in regard with the threshold 1E-6

The previous example is considered in which the point estimate of each MOV failure probability is 2.1E-4 and the error factor is 8.0. In the case of this point estimate considered as the mean value of a log-normal distribution, the 95% percentile of output system unavailability is $r^{(1)}_{0.95} = 5.64E-7 < 1E-6$ as expected in subsection *A*. On the other hand, in the case of this point estimate considered as median value, the 95% percentile of output system unavailability is $r^{(2)}_{0.95} = 2.8E-6 > 1E-6$. As shown in the Figure 2, we have only 3% in the first case but 10.72% in the second case that output system unavailability exceeds the threshold $\theta = 1E-6$. As a result, the choice, for an input uncertain variable, to interpret the point estimate value as a mean value rather than a median one leads to very different result and may have a significant impact on final decision. The difference between results in those two cases becomes more significant when the error factor is high.

In this section, we have seen that, by choosing a particular probability distribution (i.e. a log-normal one whose mean or median is considered as the point value estimated from operating experience feedback) in uncertainty analysis, the results in terms of decision making can be very ambiguous and erroneous. Moreover, it is not straightforward to define the form of a specified probabilistic distribution for non observable quantities like input parameters in PRA even if applying the expert's elicitation process. To overcome this problem, it is more appropriate to use families of probability distributions for imprecise information rather than using a single probabilistic distribution [2][3]. Such families can be obtained by resorting to probability boxes or possibility distributions or by belief functions. A comparison and formal links existing between the possibility theory, imprecise probability and belief functions have been studied in [3][10] to represent imprecise information. By using the results in [3][10], this article proposes to adopt the Dempster-Shafer theory for uncertainty analysis in PRA model according to the available data information. This theory is chosen because it can be considered as a generalization of the classical probability theory and even of possibility theory which allows a flexible way of uncertainty representation for different nature data sources. Furthermore, Dempster-Shafer calculations can use much of the probabilistic propagation framework that exists in practical PRA model.

III. DEMPSTER-SHAFER THEORY OF EVIDENCE

The Dempster-Shafer Theory (DST) is a hybrid representation, which combines the probabilistic paradigm and the interval paradigm to a unified representation. For Ferson [10], DST over the set $\Re$ of the real numbers resembles to discrete probability distribution except that the locations at which the probability mass resides are sets of real values, rather than precise points. These sets associated with non null mass are called focal elements. The correspondence of probability masses associated with the focal elements is called the basis belief assignment (BBA), noted *m*. This is analogous to the probability mass function for an ordinary discrete probability distribution. In some publications, this term is also called basis probability assignment [10] which can mislead to the assumption that *m* might be strictly a probability. The probability theory as well as the evidence theory offers either an objective or a subjective point of view of knowledge [11]. In this paper, the term *belief* is preferred because the uncertainty quantification is mainly involved in the assessment of degree of belief of experts with regard to uncertain information. In DST, this BBA on the real line is a mapping $m: 2^{\Re} \to [0,1]$ where $m(\emptyset)=0$ and $\sum_{A\subseteq\Re} m(A)=1$, for all subsets A of $\Re$. Any subset A that satisfies $m(A)>0$ is a focal element of BBA m over $\Re$. Unlike a discrete probability distribution where the mass is concentrated at distinct points, the focal elements in DST may overlap one another. Associated with each BBA are two functions *Bel, Pl* which are referred to as belief and plausibility functions. The belief and plausibility functions of uncertain variable x on interval $[\underline{a},\overline{a}]\subset\Re$ are defined as:

$$Bel(x\in[\underline{a},\overline{a}]) = \sum_{A_i\subseteq[\underline{a},\overline{a}]} m(A_i) \qquad (2)$$

$$Pl(x\in[\underline{a},\overline{a}]) = \sum_{A_i\cap[\underline{a},\overline{a}]} m(A_i) \qquad (3)$$

In the perspective of the *imprecise probabilities theory* [11], the plausibility function *Pl(A)* and belief function *Bel(A)* are upper and lower probabilities for any subset $A\subseteq\Re$. These functions bound on all possible cumulative distributions according to the given BBA. When the focal elements are reduced to singletons, the previous functions coincide with the cumulative distribution function in probability theory. In the following section, we will see how these two functions are used to represent the parameter uncertainty according to partial information through the notion of p-box.

IV. PARAMETER UNCERTAINTY REPRESENTATION WITH THE THEORY OF EVIDENCE

In the Dempster-Shafer approach, the representation of the information on the epistemic uncertain variables relies on focal elements and their basic belief assignment. In many risk applications, the data is often provided in a way that a most likely value of the variable (best estimate) associated with the confidence interval is given, for example, [lower, best estimate, upper]. Ferson *et al* [10] have developed uncertainty representations by probability bounds (also called p-box) linked to DST according to these kinds of information. The following representations are different from one another depending on the way that expert considers the best estimate as the mean or the median of epistemic uncertain variables.

- First case: The best estimate is considered as the mean

When an expert supplies the mean and the support [lower, upper] of (epistemic or aleatory) uncertain variable, we can build a family of distributions for the parameter. According to [3] only p-boxes seem to capture information in a reasonable

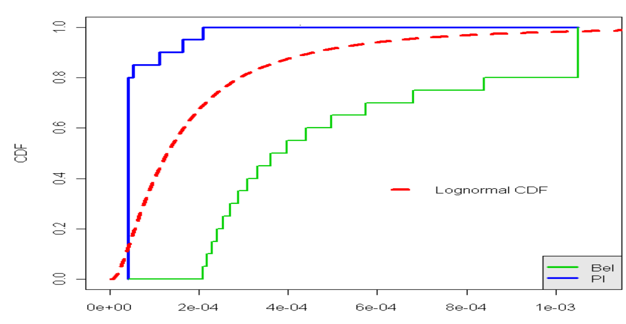

Figure 3 . *Pl/ Bel* functions with given mean and support vs. CDF of log-normal

way. However, in their opinion, the mean value does not seem to bring much information on the distribution, and the problem of finding a better, tighter representation of this kind of information remains open. Moreover, while the average value is very easy and often natural to compute from statistical data, it is not clear that this value is cognitively plausible, that is, one may doubt that a single representative value of an unknown quantity provided by an expert refers to the mean value. Figure 3 presents a p-box of a component failure probability in which the support is [4.2E-5, 1.05E-3] and the point estimate is 2.1E-04. This p-box encodes a family of probabilistic distributions with the given support and the mean value. The upper probability bound (respectively lower bound) of this p-box can be approximately seen as plausibility function (respectively belief function). The Dempster-Shafer focal elements within can be then obtained from this p-box by using a canonical discretization with N equiprobable thin rectangles or slivers [10].

- *Second case: The best estimate is considered as the median*

In this case, the median and the support are provided. This knowledge can be exactly represented by focal elements such that $m([lower, med]) = m([med, upper]) = 0.5$ where median value is noted *med*. Figure 4 presents the belief and plausibility functions of the component failure probability for this kind of information. This p-box encodes a family of probabilistic distributions with the given support and median value.

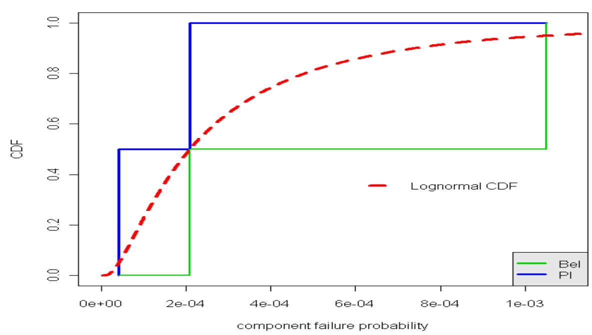

Figure 4. *Pl/ Bel* functions with given median and support

In this section, we have seen that two different interpretations of best estimate give different Dempster-Shafer representations which could lead to different results in decision making. In [3], the authors argue that the mode may best correspond to the notion of best estimate, as being the most frequently observed value or the most likely value. In this paper, we only use the two previous interpretations which are widely used in PRA community to propose an aggregated Dempster-Shafer representation as shown in the next section.

V. UNCERTAINTY REPRESENTATION IN PRA

In recent years, the use of DST to represent the uncertainties has been studied in risk analysis. In [13] the DST is applied in the context of risk analysis in drinking water treatment where the belief function is used to encode input data given by fuzzy values, interval-valued probabilities and statistical data. In the reference works [14][15], the DST replaces the probabilistic calculation on the Boolean model underlying the fault and event trees. By doing this, the uncertainty model is not built on the failure probabilities, but on the component states themselves. Recently, Dugra Rao *et al* [16] have studied the use of probability bounds approach on level-1 PRA. In their approach the uncertainties associated with input parameters are characterized by log-normal distributions in which mean values and standard deviations are given in term of intervals. By this way, we get a family of log-normal distributions that represents the uncertainty on the mean and the standard deviations. Consequently, this representation does not take into account the uncertainty on the use of log-normal distribution. Further, the input data presented in their case study, where standard deviations are given in terms of intervals, are not compatible to those provided in most of PRA data books. Indeed, for each failure event probability, the PRA data books of NPP often provides the best estimate value and the 90% confidence interval or an error factor instead. In such a context, this paper proposes to use the Dempster-Shafer representation previously presented to deal with the uncertainty on failure probabilities without relying on specified probability distribution or modifying the Boolean calculation of event/fault tree in PRA model. By using the Dempster-Shafer representation, the support can be estimated by using the concept of "error factor" or the 5% and 95% percentiles directly as done with fuzzy approach [17][18] and the best estimate can be regarded as mean or median. However, by doing so, we admit that the support contains only the information within a 90% confidence interval and therefore completely neglect the possibility to have values lower than the 5% percentile or higher than the 95% percentile of the original log-normal probability distributions.

As early mentioned, since there are two ways of interpretation of point estimate for input parameter uncertainty representation in PRA model, in this section we propose to consider these two ways of interpretation as two different Dempster-Shafer representations given by two experts. DST has an advantage in natural way of combining different sources of evidence and many methods have been proposed to construct a aggregated Dempster-Shafer representation. In [19], P. Limbourg has pointed out the advantage and robustness of the method of weighted mixing proposed by Ferson in [11] for merging different Dempster- Shafer structures. This method can be seen as the well known "linear opinion pool" algorithm for aggregation in probability theory if an uncertain parameter is represented by different probability distributions [20]. However as discussed in section 2, relying on a specified probability distribution for parameter uncertainty representation in PRA is not recommended. Therefore, the appropriate approach we propose is to define a (aggregated) unified Dempster-Shafer representation using the weighted mixing algorithm as shown on the figure 5. As can be seen, this representation contains as well the aggregated

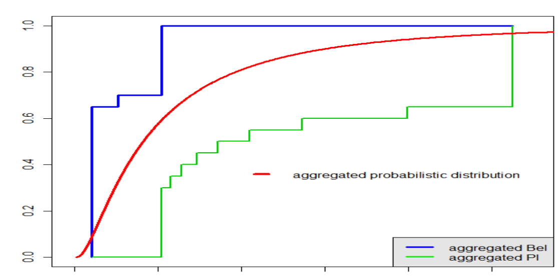

Figure 5. *Pl/ Bel* functions of the unified (aggregated) Dempster - Shafer representation

probabilistic distribution of two log-normal distributions in which the point estimate are considered as mean or median. On the one hand, this unified Dempster-Shafer representation allows to characterize the uncertainty associated to input parameters by a family of probability distributions without relying on the choice of a particular kind of probability distribution. On the other hand, this representation includes also the uncertainty due to the ambiguous interpretations of the point estimate. As a result, the output result carrying out with this representation can be considered as objective, thus used for final decision making. In the next section, we will illustrate the use of this representation through a practical example in NNP PRA model.

VI. PRACTICAL EXEMPLE

A. Problem situation

In order to illustrate the application of theory of evidence approach with the proposed unified Dempster-Shafer representation in PRA model for decision making, we implement it on one of applications of level 1 PRA in NPP at EDF: the precursor events analysis. This application consists of studying the increment of a risk metric (core damage frequency) when an event challenging the safety occurs at the nuclear plant. The risk metric is expressed as a function of input parameters such as the frequency of initiating event, component failure rate or the human error probability…The potential risk increase indicator (IRP) is used and can be expressed as the following:

$$IRP = \frac{t}{T} \times (CDF_{i,1} - CDF) \quad (4)$$

where *CDF* is the baseline risk and $CDF_{i,1}$ is the risk given that basic event (BE_i) is certain (failure probability=1). The unavailability duration of the considered basic event (*t*) within the annual duration of operation of plant (*T*) are not subjected to uncertainty in this study.

Any event that has an *IRP* greater than the threshold (1E-6) will be considered as a precursor event. In this example, to simplify the calculations, we choose an accidental sequence in an event tree (associated to the initiating event: loss of one 6,6kV board on normal shutdown mode of plant) which leads to core damage. The event considered in this paper is the failure on demand of a turbine driven pump of the auxiliary feedwater system. The duration of this unavailability is 72h within the annual duration of normal shutdown mode (T=236h). The PSA software used by EDF (Risk Spectrum® [21]) provides 89 minimal cut sets (MCS) for baseline risk *R* and 31 MCS for the risk $R_{i,1}$. We identified 30 parameters involved in the IRP calculation and among them, 28 parameters associated with uncertainty. The IRP point estimate calculated in Risk Spectrum is 2.17E-7 < 1E-6 which shows that this event is not a precursor one.

B. Uncertainty representation

The point estimates and error factors of parameters which are involved in *IRP* calculation are provided by EDF data books. For uncertainty Dempster-Shafer representation, the support of each input parameter is estimated by using the concept of error factor in which the lower bound = point estimate/ EF and the upper bound = point estimate * EF. The belief and plausibility functions of each parameter are calculated from the unified Dempster-Shafer representation using the weighted mixture aggregation as depicted in section 5.

C. Uncertainty propagation

Propagating uncertainties specified in the Dempster-Shafer framework through system functions consists of propagating focal elements. However, the technique of propagating is not as straightforward as for the probabilistic approach. As discussed in [19], being a synthesis between probabilistic and interval arithmetic, DST relies on optimization to propagate focal elements through the system function. Given a focal element from the joint distribution, the mass is propagated through the system function. Thus, the propagation of a focal element involves the solution of two optimization problems (min, max). The propagation speed therefore heavily depends on the 'well-behaving' of the function and the required accuracy. The IRP or other indicators in PRA applications are often functions whose monotonicity properties with respect to input parameters are usually unknown. It is therefore necessary to use complex optimization techniques.

D. Results and interpretations

The table 1 shows resulting statistics of IRP output by using Monte Carlo sampling with 50000 samples in the probabilistic approach which is used in current PRA practice. The second column presents results of the case in which the point estimate is interpreted as the mean value of a log-normal distribution and the third column presents results of the case in which point estimate is interpreted as the median value. We see that the percentiles of *IRP* in former case are smaller than that of later. Moreover, in the first case where we have only 3.8% chance that *IRP* exceeds the threshold $\theta = 1E-6$, decision makers can think that the event may not be a precursor event. However, in the second case where we have a very higher percentage (19.9 %), the final decision could be different. In both cases, the final decision can not be surely made because of the unjustified assumption of log-normal distribution.

Table 1. Statistics on the resulting IRP output in probabilistic case

	Point estimate as mean	Point estimate as median
Mean	2.167E-7	1.055E-6
Median	5.754E-8	2.438E-7
5% percentile	4.632E-9	1.778E-8
95% percentile	8.226E-7	4.001E-6
Pr(IRP>1E-6)	0.038	0.199

The table 2 shows resulting statistics of IRP output when using the unified Dempster-Shafer representation obtained from the aggregation of two interpretations of point estimate without relying on any specified probabilistic distributions. In order to implement the Dempster-Shafer framework for a PRA model, we used the imprecise probability propagation package for R programming language. This is a free open source package outlined in [22]. The resulting plausibility and belief functions of IRP are shown in the Figure 6. Unlike the probabilistic approach where output results are represented by single values, the results in Dempster-Shafer approach are given in terms of intervals. The expectation values and mean values are approximatively the same level.

Table 2. Statistics on the resulting IRP output with DST

Mean	[1.829E-7 , 1.513E-6]
Median	[1.086E-7 , 4.148E-7]
5% percentile	[8.014E-9, 8.305E-8]
95% percentile	[5.552E-7, 1.285E-5]
Pr(IRP>1E-6)	[0.000 , 0.278]

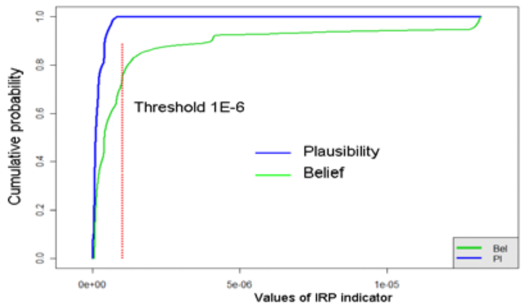

Figure 6. Pl/Bel function of IRP output with (unified) aggregated Dempster -Shafer representation

The probability that *IRP* exceeds the threshold (1E-6) lies in the interval [0.000, 0.278] which contains also those of previous probabilistic cases. This means that in worst case we have 27.8% chance that the basic event (failure on demand of a turbine driven pump) is considered as a precursor event while in the best case, this event is considered as a non precursor event. Since the output results have been taken into account the uncertainty associated with the unjustified assumption of log-normal distribution and the uncertainty due to the ambiguous interpretations of the point estimate, these results can be surely employed in the final decision making process.

VII. CONCLUSIONS AND PERSPECTIVES

In this paper, we have shown that two different interpretations of point estimate for input parameter may have a great impact on decision making in PRA application. Moreover, since the use of a log-normal distribution is not properly justified, it would be better not to make the assumption of a single probabilistic distribution. The Dempster-Shafer theory is studied as an alternative approach to characterize the parameter uncertainty in a more appropriate manner. The so-called unified Dempster-Shafer representation is proposed in this paper as an attractive way which allows us to deal with current issues due to the unjustified assumption on the log-normal and the ambiguous interpretations of point estimate value. A practical example is used to illustrate the use of this representation in PRA model. The results have shown that even though the output results with this unified Dempster-Shafer representation seems to be more conservative in comparison with the probabilistic approach, these results can be surely used for further decision making because it allows decision makers to take into account the parameter uncertainty in a proper way and to have further information about the final results in the best case and worst case without having to bet on the form of a single probabilistic distribution.

However, since the Dempster-Shafer approach has just been recently studied in probabilistic risk assessment, the use of this approach must be considered in further study and needs a review of PRA experts' opinions to be operational. Another problem which needs to be noticed is that the final results given in term of intervals in the DST are not well known for decision makers, which makes the decision process more difficult. These aspects will be considered in further works.

REFERENCES

[1] M. Drouin, G. Parry, J. Lehner, G. Martinez-Guridi, J. LaChance, T. Wheeler, *Guidance on the Treatment of Uncertainties Associated with PRAs in Risk-informed Decision making*, Main Report, NUREG1855-V.1, 2009.

[2] J.C Helton and W. L. Oberkampf, *Alternative Representations of Epistemic Uncertainty*, Special Issue of Reliability Engineering and System Safety, Vol. 85, 2004.

[3] C. Baudrit and D. Dubois, *Practical Representations of Incomplete Probabilistic Knowledge*, Computational statistics & data analysis, Vol. 51, n° 1, pp. 86-108, 2006.

[4] J. C. Helton, J. D. Johnson, W. L. Oberkampf, C.J. Sallaberry, *Representation of Analysis Results Involving Aleatory and Epistemic Uncertainty*, Sandia National Laboratories, SAND2008-4379,

[5] Electric Power Research Institute (EPRI), *Treatment of Parameter and Modeling Uncertainty for Probabilistic Risk Assessments*, Final Report, December 2008

[6] A. Villemeur, *Sûreté de fonctionnement des systèmes industriel*s, Eyrolles & EDF Editeurs, 1988.

[7] G. Apostolakis and S. Kaplan, *Pitfalls in Risk Calculations,* Reliability Engineering Vol 2,pp.135-145, 1981

[8] T.D Le Duy, *Traitement des incertitudes dans le contexte des applications des EPS*, master thesis, EDF-UTT, 2008.

[9] S. A. Eide, *Historical perspective on failure rates for US commercial reactor components,* Reliability Engineering System Safety, Vol. 80, pp.123-132 May 2003,

[10] S. Ferson, L. Ginzburg, V. Kreinovich, D. Myers and K. Sentz, *Construction of Probability Boxes and Dempster-Shafer structures*. Sandia National Laboratories, Technical report SAND, 2002-4015, 2003.

[11] C.Simon, P.Weber, *Imprecise reliability by evidential networks*, Journal of Risk and Reliability 223, pp.119-131, 2009.

[12] P. Walley, *Statistical Reasoning with Imprecise Probabilities*, Chapman & Hall, London, 2001.

[13] S. Démotier, W. Schön T. Denoeux, *Risk Assessment Based on Weak Information using Belief Functions: A Case Study in Water Treatment*, IEEE, Transactions on Systems, Man and Cybernetics C, Vol.36, Issue 3, 382-396, 2006.

[14] M.A.S. Guth, *A probabilistic foundation for vagueness and imprecision in fault-tree analysis*, IEEE Trans Reliability, 40(5), pp.563–571, 1991

[15] F. Tonon, A. Bernardini and A. Mammino, *Determination of parameters range in rock engineering by means of random set theory*, Reliability Engineering System Safety, 70(3), pp.241–261, 2000.

[16] K. Durga Rao, H.S Kushwaha, A.Kvermaa and A.Srividya, *Uncertainty analysis based on probability bounds (P-box) approach in Probabilistic Safety Assessment*, Risk Analyis, Vol.29, No.5, 2009.

[17] D. Huang, T. Chen, M.J.J Wang, *A fuzzy set approach for event tree analysis, Fuzzy Set and Systems*, 118, pp.153-165, 2001.

[18] P.V. Suresh, A.K. Babar, V. Venkat Raj, *Uncertainty in fault tree analysis: A fuzzy approach*, Fuzzy sets and Systems, 83, pp.135-141, 1996

[19] P. Limbourg, R. Savić, J. Petersen, H.-D. Kochs, *Fault Tree Analysis in an Early Design Stage using the Dempster-Shafer Theory of Evidence,* European Safety and Reliability Conference, ESREL 2007, pp. 713-722.

[20] M. Stone, *The opinion pool*, Annals of Mathematical Statistics 32: 1339-1342, 1961.

[21] AB Relcon, *Risk spectrum professional-theory manual*, 2004.

[22] P. Limbourg, *IPP Toolbox V1.0* (a R package for uncertainty quantification and propagation in the framework of Dempster-Shafer Theory & imprecise probabilities), 2008. http://cran.r-project.org/web/packages/ipptoolbox/index.html