

Optimal transport under Dempster Shafer Theory

Barbara Vantaggi¹[0000–0002–3651–3743]

Sapienza University of Rome barbara.vantaggi@uniroma1.it

Abstract. The optimal transport problem has gained significant attention in the probabilistic contexts in the last decades, it relies on the probabilistic marginal problem and its popularity is closely linked to the Wasserstein distance [3].

The optimal transport problem and so the marginal problem is studied by relaxing the requirement of completely specified marginal probabilities and referring to belief functions, that reveals to be a sufficiently general framework to model this kind of uncertain knowledge.

This allows to define the optimal transport problem in Dempster-Shafer theory, for which several definitions can be given [1, 2, 5].

A suitable set of bivariate joint belief functions having fixed marginals is then analysed and is used to minimize a functional of a given cost function, so as to select an optimal imprecise transport plan in the form of a joint belief function.

We formulate two Kantorovich-like optimal transport problems by seeking to minimize the Choquet integral of the cost function with respect to either the reference set of joint belief functions or their dual plausibility functions.

Our pessimistic and optimistic Kantorovich-like optimal transport problems can be expressed in terms of a particular class of joint Möbius inverses. Then, by suitably extending the cost function on Cartesian products of marginal focal elements, we fall in the formulation of optimal transport problem given in [1].

We give an application by choosing a metric as cost function: this permits to define pessimistic and optimistic Choquet-Wasserstein pseudo-distances, that can be used to compare belief functions on the same space.

Next, the formulation of optimal transport is generalized by allowing different behaviours parameterized by a pessimism index $\alpha \in [0, 1]$.

We consider the problem of finding optimal joint belief functions whose Möbius inverse is “sparse” (with a minimal set of focal elements). To this aim, we add a suitable regularization term and provide a characterization of the unique optimizer of the regularized problem, in terms of solutions of its dual problem. Two regularization schemes that enforce two different forms of sparsity are introduced: quadratic regularization seeks global sparsity, while group lasso regularization refers to groups of focal elements of one of the two marginals.

This setting allows us to model a game under ambiguity, through the definition of the Stackelberg-Cournot-Nash equilibrium with belief functions and α -maxmin preferences.

Finally, a distinguished application is found in a market with a finite set of types of investors having misspecified distribution.

Investors' choices result in a purchase portfolio on a finite set of assets which, in turn, depends on asset prices fixed by the market maker and the interactions among investors.

Keywords: Optimal transport · Belief function · Stackelbeg-Cournot-Nash equilibrium.

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